Comparison of Main Models for Size Effect on Shear Strength of Reinforced and Prestressed Concrete Beams

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Abstract

This paper presents a critical comparison of the existing code provisions for shear strength of concrete beams. The comparison is based on the interpretation of the available multivariate database on shear strength, the examination of the predicted size effects on shear strength and their underlying hypotheses, and the results of recent high-fidelity numerical simulations of shear failure. In addition to examining the existing models, the present comparison also provides several key considerations for testing the scientific soundness of any model of shear failure of concrete beams, which is necessary for future revisions of the design code provisions.

Keywords

Shear strength; size effect; reinforced concrete; fracture energy; multivariate database

1 Introduction

Among the design codes of three major engineering societies, including ACI, *fib* and JSCE [1][2][3]**,** JSCE was in the early 1980s the first to introduce for shear strength of beams a size effect, albeit of a statistical form known today as inapplicable. CEB, the predecessor of *fib*, was the first to recognize that the size effect was not statistical, but the empirical form introduced is known today as unrealistic. ACI remains the only society that does not yet take the size effect on the shear strength of concrete beams into account. Yet ACI is also the only one among the three that does not have in its code a size effect that is questionable from today's perspective, and thus has a chance to be the first society to introduce a scientifically sound form (note, though, that all the three societies have already introduced the size effect, and one of the correct form, into the specifications for anchor pullout[1][2][4][5], which is essentially a shear failure).

The size effect is a general property of failure of all quasibrittle materials [6][7][8][9], among which concrete is an archetypical case. Currently, a debate about introducing the size effect into the specifications of ACI-318 design code [1] for the shear strength of reinforced and prestressed concrete beams is under way in committees ACI-445 and ACI-446. A similar debate about the Model Code has been initiated at the *fib* Congress in Prague in 2011[10]. The question in all societies no longer is whether the size effect should be taken into account, but what is the proper form of size effect to adopt. The aim of this brief paper is to aid this debate by explaining and critically comparing the main options and the methods of their evaluation.

2 Extracting Statistical Evidence from a Biased Heteroscedastic Database

Fig. 1 shows the newly-compiled database (ACI-445d/DAfStb), an expansion of the previous ACI 445F database [11]. The shear strength v_c contributed by the concrete is plotted versus beam depth d in a double logarithmic plot. Each point represents one laboratory test of a reinforced concrete beam without stirrups that failed due to shear. Highly scattered though the data are, they nevertheless show a downward trend. This trend tempted some engineers in ACI-445D to pass a regression line, which happens to have in the log-log plot the slope of about −1/3 and thus gives a size effect factor of the type $\theta = (d/d_0)^{-1/3}$, where d = beam depth (measured from the top face to the longitudinal steel centroid) and d_0 = constant.

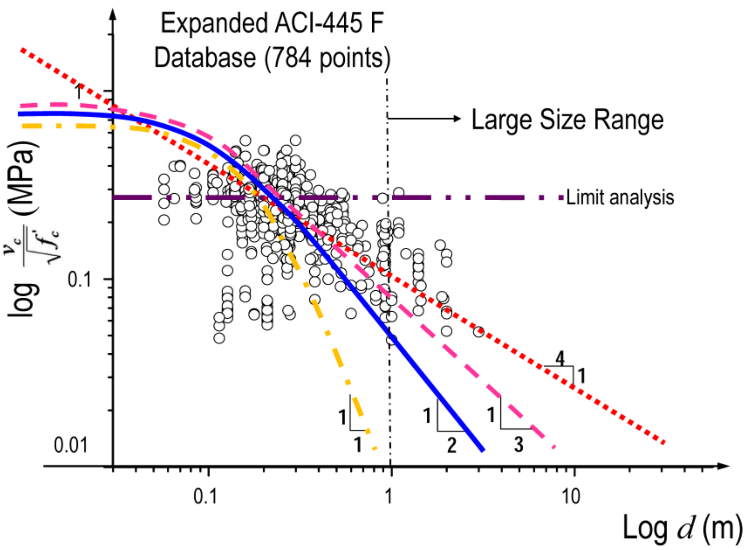

Fig. 1. Expanded ACI-445F database of 784 shear beam tests and various size effect asymptotes.

However, such a simplistic regression analysis is incorrect. One reason is that, inevitably, the database available is heavily biased statistically. The collected data points do not cover the size range uniformly (i.e., the database is not homoscedastic). In the existing database, plotted in Fig. **2**, 87% of data points pertain to beam depth d < 508 mm (20 in.) and 97% to beam depth d < 1.27 m (50 in.), while nowadays the outriggers of tall buildings have the depth of about 6 m (19.7 ft.) and bridge girder in Palau, which collapsed by shear with compression, had the depth of 14.2 m (46 ft.) [12][13].

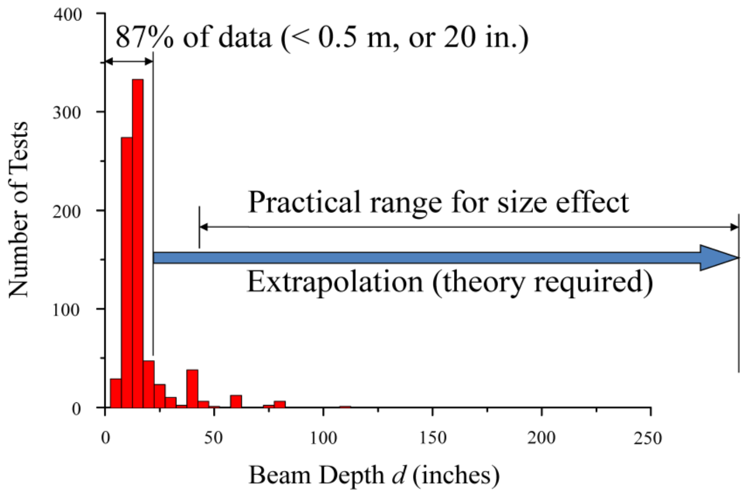

Fig. 2. Histogram of number of beams tested versus beam depth in inches.

Another source of bias, in fact a major one, is a simultaneous variation of the means of steel ratio ρ_w, shear span ratio *a/d* and other secondary variables as *d* increases. This is due to the fact that the database has not been (and could not have been) generated according to a proper statistical sampling scheme. E.g., if the size range is subdivided into several intervals of constant width in logarithmic scale (five in Fig. **3**), the averages of steel ratio ρ_w and shear span ratio *a/d* (as well as of concrete strength f_c' and maximum aggregate size d_a), calculated separately for each size interval, should be about the same for all the size intervals [14]. But, as seen in Fig. **3** (top), both the averages of ρ_w and *a/d* in the subsequent size intervals decrease with increasing size. For ρ_w, they decrease by an order of magnitude, and for *a/d* by about 30% (as it happened, the testers chose for large beams generally a smaller ρ_w and a smaller *a/d* than they did for small beams). Same kind of statistical bias is found for f_c' and d_a; see Fig. **3** (bottom) (in which $a/d = M/V\,d$ where M, V = bending moment and shear force at the shear critical cross section, respectively).

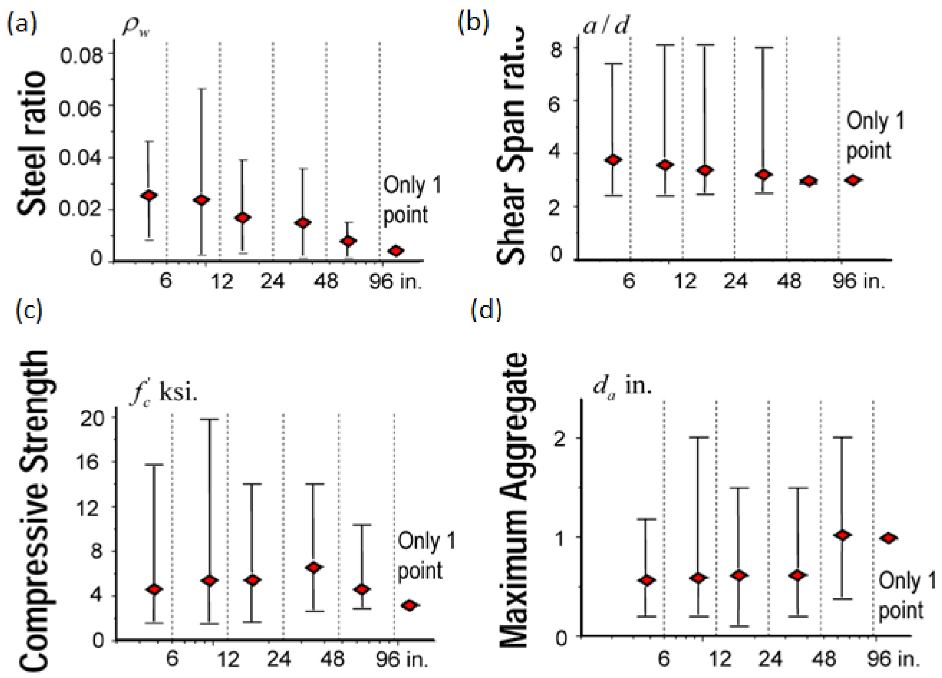

Fig. 3. Variation of secondary parameters in different size intervals: a) longitudinal steel ratio ρ_W; b) shear span ratio *a/d*; c) concrete strength; and d) maximum aggregate size d_a.

2.1 Filtering of Database to Remove Bias in Secondary Variables

In previous work [14], a computer program was developed to delete gradually, in an unbiased way (without human intervention), the extreme points in each size interval, so as to achieve a nearly uniform variation of the secondary parameters throughout the size intervals. This program deletes outlier points one by one, selecting each point deletion candidate so as to achieve the greatest possible reduction of the variance of the averages in all the intervals. Here this program is used to handle the enlarged database of over 700 data points. In a database filtered in this way, with the number of points reduced by computer filtering, shows the values of average $\bar{\rho}_w$, $\overline{a/d}$, and $\bar{d}_a$ to be almost uniform throughout the size intervals. Over ten unbiased filtered databases were generated by the computer, with different sets of averages $\bar{\rho}_w$, $\overline{a/d}$, and $\bar{d}_a$, and with different sets of undeleted points. To save space, Fig. 4 shows only 6 of them, along with the corresponding regressions of optimum fit of the average values of shear strength (circles) of the remaining data points (crosses) in each size interval. The trend is seen to be very different from the regression of the unfiltered database in Fig. **1.** The averages of the data in the size intervals now agree closely with the ACI-446 size effect formula, which terminates with the size effect factor

$$\theta_{LEFM} \approx \sqrt{\frac{d_0}{d}} \tag{1}$$

where d_0 is a constant for geometrically similar beams. This size effect factor is characteristic of fracture mechanics of sharp cracks (i.e., linear elastic fracture mechanics, or LEFM). In the log-log plot, the terminal slope is −1⁄2; see Fig. 4. Further note that if the average of data in each interval is taken with the same weight, the bias due to unequal numbers of data points in subsequent intervals also gets eliminated.

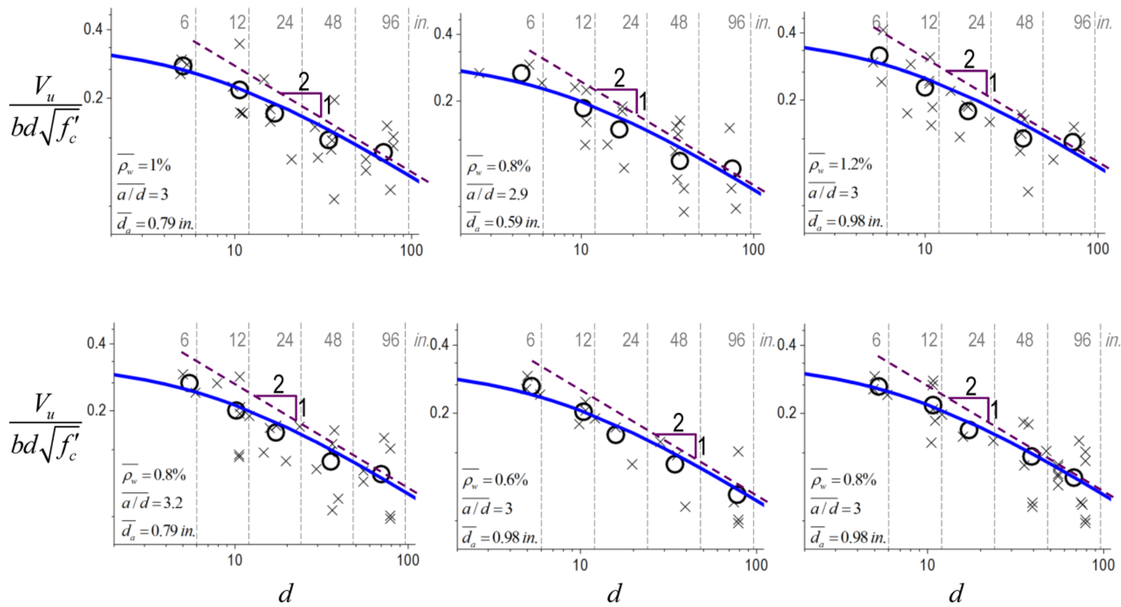

Fig. 4. Data filtering to achieve uniform distribution of secondary parameters throughout size intervals and the fitting based on the mean values of remaining test points.

2.2 Evidence from Weighted Multivariate Regression of Complete Database

Another way to cope with the database bias (or heteroscedasticity) is to attach to the data points weights inversely proportional to the data density in transformed variables (i.e., in log *d*), and then use multivariate nonlinear optimization of the data fits. This can be done by a standard computer subroutine, such as the Levenberg-

Marquardt algorithm, which optimizes all the variables simultaneously [15][16], and also delivers the coefficients of variation of the optimized parameters. This is a standard procedure for dealing with a heteroscedastic data set [17], which is the case here.

The bias due to size dependence of data density can be characterized by the numbers, N_i , of data points in the subsequent size intervals, which are here found to decrease sharply with increasing size d. If this bias were ignored, the resulting optimum fit of data would be dominated by small beams, particularly those with d < 508 mm (20 in.), while the data for beams with d > 1.27 m (50 in.) would have hardly any effect on the optimum fit. This statistical bias must be countered by attaching weights to the data points.

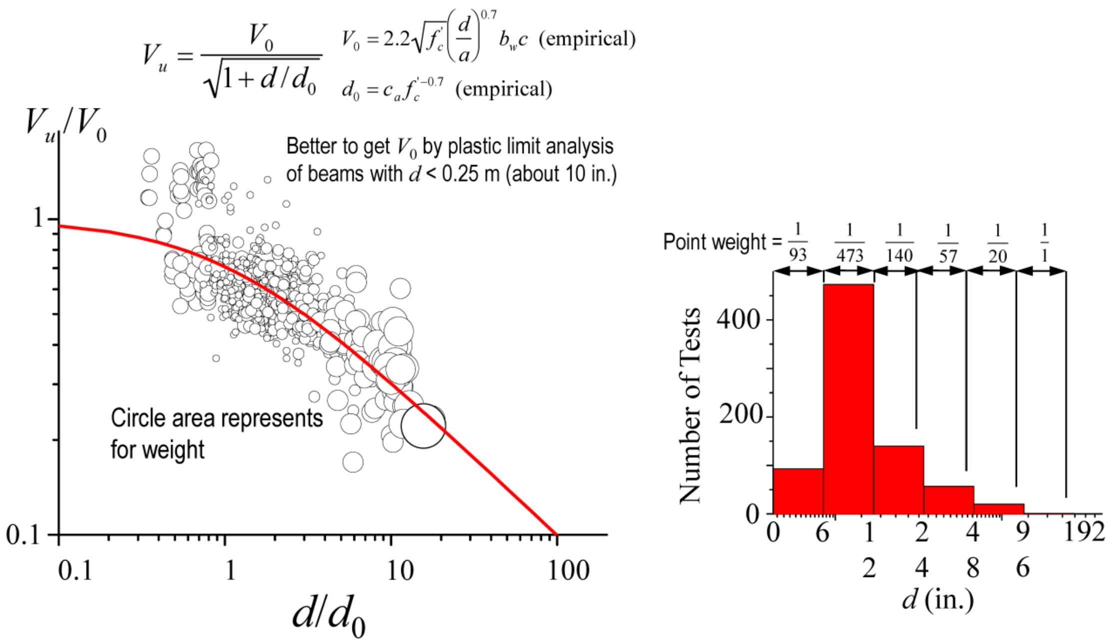

Fig. 5. Fitting of the expanded ACI-445D database of 784 weighted points .

To this end, one needs to subdivide the $\log d$ scale into several equal size intervals i = 1, 2, ...n_i, then count the number N_i of the data points in each size interval, and finally apply to all the data points in interval i the weight $1/N_i$ [15][16] (similarly, further weights could be attached to the data points in multi-dimensional boxes in the space of all secondary variables, to counter the nonuniformity of the number of points in the boxes). An optimum fit of the shear database obtained in this manner is shown in Fig. **5,** in which the solid curve is the optimum fit by the formula of ACI committee 446 and the areas of the circular data points are proportional to their weights, for better visualization.

Normally, multivariate regression with proper weighting would be sufficient to analyze the database. However, to succeed, the form of the dependence of v_c on concrete strength and the composition parameters of concrete beam would have to be known well. Unfortunately, it is not. This is why the creation of filtered unbiased subsets of the database is useful.

3 Load Capacity in Shear

In elasticity with strength limit as well as plastic limit analysis, the characteristic (or nominal) stress at maximum load (or at failure under load control), the nominal strength (e.g., v_u in the case of beam shear), is independent of the structure size, d, when geometrically similar situations are compared. The dependence of this strength on the structure size, d, cannot be captured by elasticity or plasticity and came to be called the size effect. Since the shear force V_u carried at maximum load by the concrete is (at constant beam width) proportional to $v_u d$, V_u increases linearly with the structure size when the size effect is absent, but slower than linearly when it is present (for example, such size effect has long been embodied in the ACI specifications for anchor pullout [1][4][5]).

Fig. **6** shows four plots of V_u versus beam depth d, corresponding to the ACI code (ACI-318-14 [1]), to Model Code 2010 of *fib* [2], to the code of JSCE (Japan Society of Civil Engineers) [3], and to the 2007 code proposal of ACI Committee 446 [18]. For comparison, all the curves are scaled to the same initial slope. All the asymptotic size effects on V_u are power laws of d, as indicated in the figure, and the size effect formulas are also listed. The differences among the size effect curves are certainly striking and are discussed next.

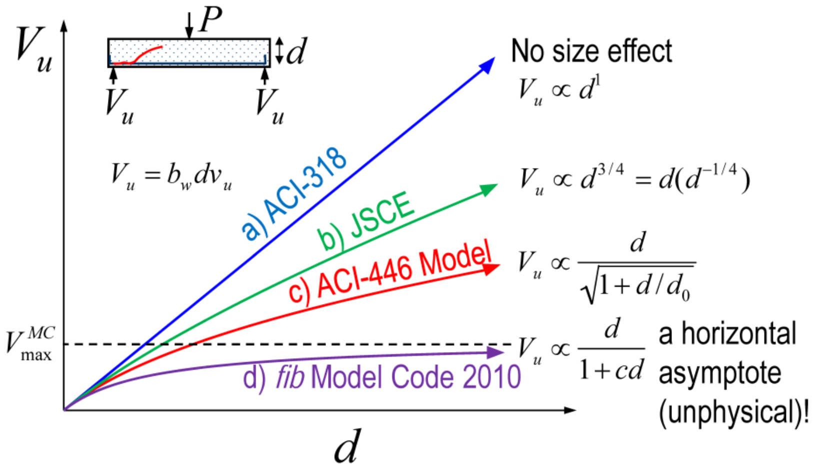

Fig. 6. Comparison of the load capacity asymptotes based on different size effects.

3.1 Size effect of JSCE

Consider first the JSCE curve. The JSCE was the first society to introduce the size effect into its design code in the early 1980s, based on the vision of H. Okamura [19][20]. It was a revolutionary step. Even though the JSCE formula for size effect is today known not to be the correct one, it nevertheless provides significantly better structural safety than ignoring the size effect altogether. Of course, in the early 1980s, JSCE could not introduce a better formula because the quasibrittle fracture mechanics, required for concrete, had not yet been developed. The only size effect theory that existed in the early 1980s was the Weibull statistical theory [21], according to which

$$v_u = v_0 \theta_{JSCE} \,,$$

$$\theta_{JSCE} = \left(\frac{d_0}{d} \right)^n . \tag{2}$$

where d_0 = experimental constant for beams of similar shape. Based on some experiments for Weibull theory prior to 1980, exponent n was thought to be about 1/4 for concrete, although based on the current calibration of Weibull theory for concrete, n should be about 1/12.

Today, however, it is clear that the Weibull statistics, implied by the JSCE, does not apply to beam shear, and so the JSCE size effect has no valid theoretical foundation. Applicability of the Weibull size effect rests on the weakest-link model, which assumes the existence of many material elements (or representative volume elements, RVEs) such that the failure of only one of them triggers failure of the whole structure. The larger the structure, the more failure-triggering elements exist, and since the material strength is a random field, the strength of the weakest element decreases with the number of elements, and thus also with the structure size.

Such a situation exists in some unreinforced concrete structures, e.g., arch dams, but not in the shear failure of beams. The reason is that the location of the point of failure initiation is predetermined by mechanics. The failure initiates at the tip of a long diagonal shear crack that forms before the maximum load. The relative location of this tip is almost fixed by mechanics, which dictates the crack path, and is about the same regardless of structure size. This fact was established by finite element fracture analysis and is confirmed by many experiments. The best experimental confirmation is furnished by the crack paths in Fig. **7** recently observed in geometrically scaled tests of beams of very different sizes [22]. Therefore, the randomness of material strength cannot be the cause of size effect on the mean shear strength (although it surely affects the scatter).

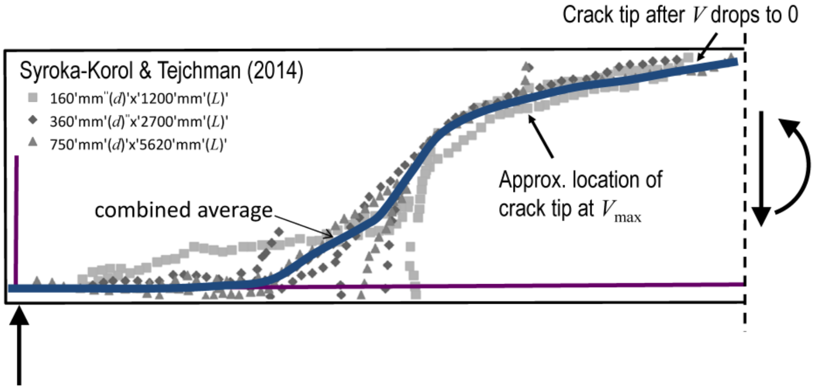

Fig. 7. Similarity of crack pattern documented in scaled beams of different sizes.

3.2 Size effect of fib Model Code 2010

Another kind of size effect curve was introduced into the *fib* Model Code (MC) 2010 [2]. It replaced an earlier purely empirical curve introduced in 1990 into the CEB Code. The MC size effect curve has the form

$$v_u = v_0 \theta_{MC} ,$$

$$\theta_{MC} = \frac{1}{1 + d / d_0} . \qquad (3)$$

where v_0 and d_0 are constants, and θ_{MC} = size effect factor of *fib* MC 2010. The size dependence of θ_{MC} is plotted in Fig. **6.** This curve gives the extreme size effect among all the formulae proposed so far.

The MC size effect factor, Eq. (3), is not correct. This may be, for example, simply demonstrated by the asymptote of the curve of $V_u = b_w d v_u$ versus d (here b_w is beam width). In contrast to all the other size effect curves, this asymptote is horizontal, featuring the only upper bound on V_u among all the size effect curves proposed so far. Thus, if a beam is sufficiently deep, the doubling of its depth, for example, would not cause any increase of load capacity. This feature obviously defies common engineering sense. A change in the Model Code 2010 is inevitable.

Why has this serious problem with Model Code 2010 been overlooked? This is probably due to two reasons: 1) the size effect has normally been plotted as the dependence of v_u rather than V_u on size d; and 2) the plot of v_u versus d has habitually been presented in the linear (rather than logarithmic) scale of d, which does not reveal the asymptotic behavior and cannot show the quality of fit outside a narrow central range of sizes.

The size effect suggested by Model Code 2010 was conceived as a generalization of the so-called modified compression field theory (MCFT) which gives v_0. This theory works well for small beams, d < 254 mm or 10 in., for which the size effect is negligible ($\theta_{MC} \approx 1$), and is acceptable for d < 508 mm or 20 in.

But why, for larger beams, the generalization of MCFT led to an incorrect size effect? — Because the generalization rested on the following unrealistic hypotheses [16]:

1. The first hypothesis was that what controlled the size effect was the spacing s_e of multiple parallel diagonal shear cracks. However, these cracks do not control failure; they form at crack initiation, well before reaching the maximum load and before localizing into one dominant diagonal crack.
2. The second was a softening law for the parallel diagonal cracks [23], which was chosen arbitrarily, in disregard of the amply verified softening stress-separation law for concrete fracture.
3. The third hypothesis was that the stress transmission across the diagonal shear cracks determines the shear load capacity due to concrete, in the manner pictured in Fig. **8**a. But later finite element failure analysis [16], based on a realistic triaxial constitutive model for damage in concrete (the microplane model), contradicted this hypothesis. It revealed that the stress transmission across the diagonal crack has, for large beams, a minor role at best. More specifically, it revealed that:
   (a) The uniformity of distribution of the aggregate interlock stresses along the diagonal shear crack, implied in the MCFT generalization, is true only for very small beams (d < 254 mm or 10 in.). For large beams, these stresses localize while the localized peak travels along the diagonal crack as the crack is being opened under increasing load (Fig. **8**b);
   (b) The stresses transmitted across the diagonal crack at maximum load

contribute in small beams about 40% of the shear force V_c carried by concrete, and in deep beams only a negligible fraction of V_c—only 23% of V_c in the beam 1.89 m (74.4 in.) deep tested in Toronto [16]). The contribution gets still smaller for larger beams.

(c) Most of the shear force due to concrete at maximum load is carried by an (imagined) inclined compression strut above the diagonal crack, loaded by inclined compression stresses transmitted across the ligament above the tip of the diagonal crack (Fig. **8**c)

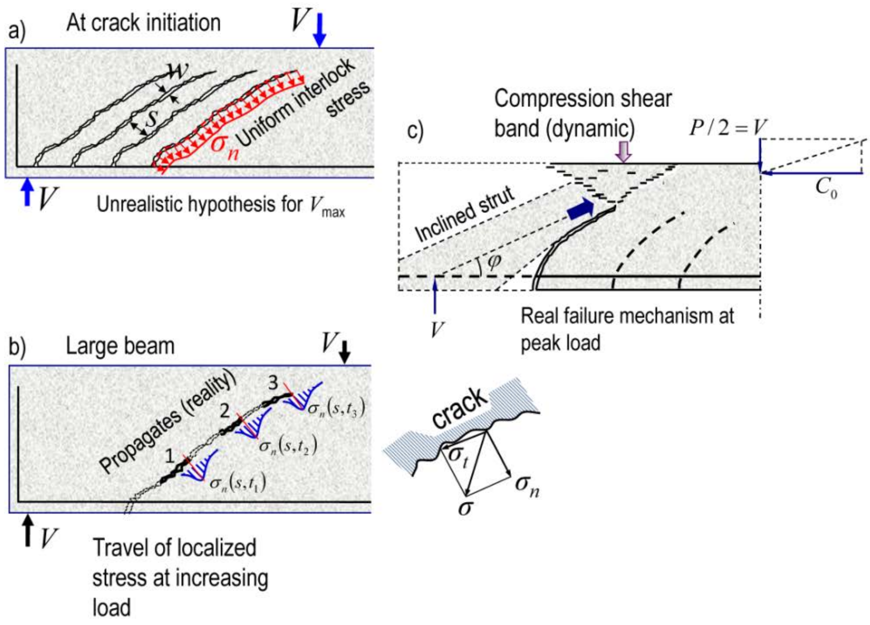

Fig. 8: a) Interlock stress distribution assumed in MCFT; b) localization of bridging stress as diagonal cracks propagate; and c) shear stress transmitted through the inclined compression strut.

3.3 Roles of Stresses across Main Diagonal Shear Crack and along Compression Strut

Figs. **9** and **10** show comparisons of Eq. 7 with the main published data on the size effect on the shear strength due to concrete in reinforced concrete beams without stirrups [24][25][26]. The optimum values of $D_0 = d_0$ are indicated and the best fits by other size effect curves are shown (note that, for beams with stirrups, the optimum values of d_0 obtained from individual size effect tests are, according to a recent study [27], about ten-times larger, which pushes the size effect into extremely deep beams). These data have been used to calibrate a finite element program featuring microplane model M7 for concrete, an advanced and realistic constitutive law for fracturing and damage in concrete (which is embedded in commercial software ATENA). It has been implemented in user's subroutine UMAT of ABAQUS, and then used to check some assumptions.

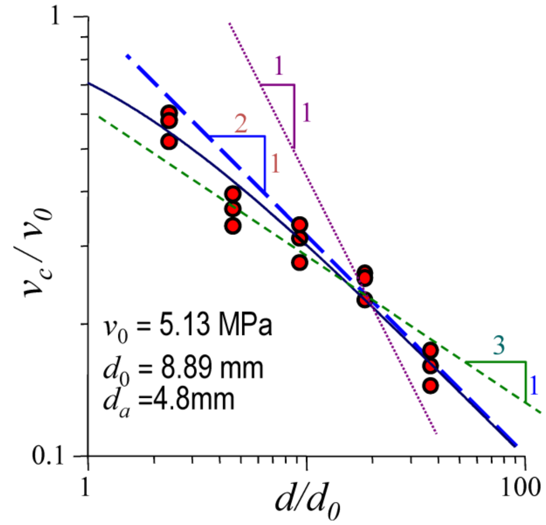

Fig. 9. Size effect tests of beam shear conducted at Northwestern University and the fittings based on different size effects.

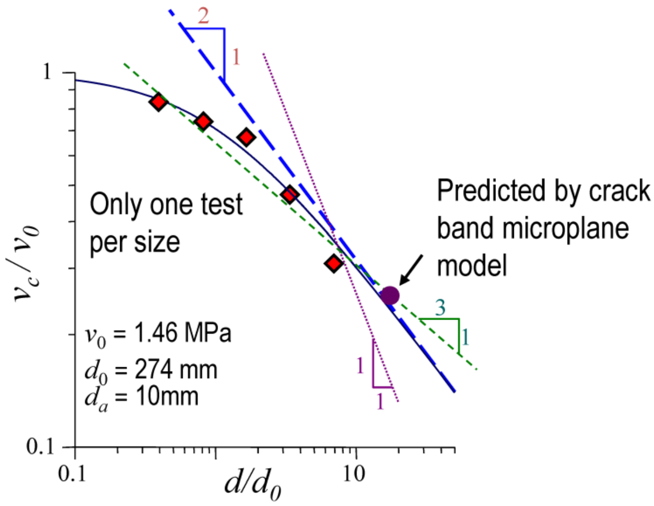

Fig. 10. Size effect tests of beam shear conducted at the University of Toronto and the fittings based on different size effects.

For example, Fig. **11** shows the stress distributions calculated for small and large beams very similar to those tested at University of Toronto. They confirm that in small beams the compression strength of concrete at maximum load gets mobilized throughout the whole ligament above the tip of the main diagonal crack while, in large beams, the stress gets localized into only a part of that ligament. This provides a simple explanation of the size effect. Also, in Fig. **10**, the program with Model M7, calibrated by the Toronto tests for five beam sizes, is used to compute an additional, 6^{th}, point for a beam of depth 5 m, as shown in Fig. **10.** Note that this calculated point agrees well with the terminal slope of −1/2 of the ACI-446 size effect at the large-size limit, Eq. 7, and disproves the terminal slope −1 of the large size asymptote implied by Model Code 2010.

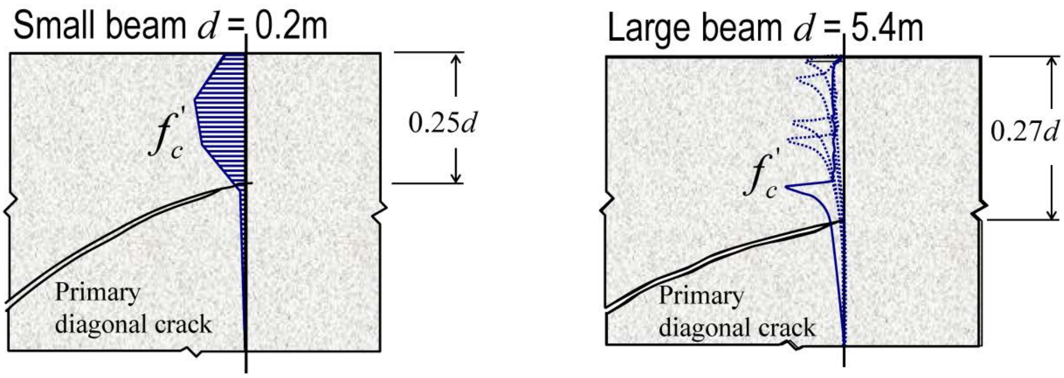

Fig. 11. Stress distributions at V_{max} along the ligament above the primary crack tip in small and large beams.

4 ACI-446 Size Effect Indicated by Dimensional Analysis and Energy Balance

This size effect, equation c) in Fig. **6**, was unanimously endorsed in 2007 by ACI committee 446, Fracture Mechanics. It is deterministic and is caused by the release of strain energy of the structure during failure (as thus it is also called the energetic size effect). It has been derived as a general consequence of fracture mechanics of quasibrittle materials for failures, called Type 2 failures, in which the maximum load is reached only after long stable crack growth [7] (Type 1 failures, which are those occurring right at crack initiation, exhibit a different type of size effect). The Type 2 failures are typical not only of reinforced concrete, but also of sea ice, fiber composites, tough ceramics, wood, rigid foams, rock masses, and all other quasibrittle materials.

These are brittle heterogenous materials in which the inhomogeneity size and the fracture process zone size are not negligible compared to the structure dimensions in engineering applications. This is a salient characteristic of concrete structures and, in particular, of beam shear.

The simplest, yet general, derivation of the size effect in quasibrittle fracture can be based on dimensional analysis and energy balance [6]. The total release of (complementary) strain energy W caused by fracture is a function of both: 1) The *length* a of the fracture (or crack band) at maximum load, and 2) the *area* of the zone damaged by fracturing, which is $w_c a$ where $w_c = n d_a$ = material constant = width of the crack band swept by the fracture process zone width during the propagation of the main crack, and n = 2 to 3, depending on the relative stiffness and strength of aggregate pieces and mortar. Parameters a and $w_c a$ are not dimensionless. But they can appear in the equation governing failure only in dimensionless form, which means that W can depend only on dimensionless parameters $\alpha_1 = a/D$ and $\alpha_2 = w_c a/D^2$, where D can be taken as either total beam depth h or as depth d from the compressive face to the reinforcement centroid. So the total strain energy release W must be written as

$$W = \frac{1}{2E}(\frac{P}{bD})^2 bD^2 f(\alpha_1, \alpha_2) \tag{4}$$

where b_w = beam width. In the case of geometrically similar beams of different sizes, f is a smooth function independent of D. The energy balance during crack propagation requires that $\partial W/\partial a = G_f b$, where G_f = critical value of the energy release rate. Noting that

$$\frac{df}{da} = \frac{\partial f}{\partial \alpha_1}\frac{\partial \alpha_1}{\partial a} + \frac{\partial f}{\partial \alpha_2}\frac{\partial \alpha_2}{\partial a} \tag{5}$$

where $\partial\alpha_1/\partial a$ = $1/D$ and $\partial\alpha_2/\partial a$ = w_c/D^2, and considering the first two linear terms of Taylor series expansion $f(\alpha_1, \alpha_2) \approx f(0, 0) + f_1\alpha_1 + f_2\alpha_2$ where $f_1 = \partial f/\partial\alpha_1$, $f_2 = \partial f/\partial\alpha_2$, one obtains the equation

$$\left(\frac{f_1}{D} + \frac{f_2\omega_c}{D^2}\right)\frac{P^2}{2bE} = G_f b \tag{6}$$

After rearrangement, and with the notation $v_u = P/b_wD$ = average (or nominal) shear strength due to concrete, Eq. 6 yields the deterministic (or energetic) size effect of ACI-446:

$$v_u = v_0\theta,$$
$$\theta = \frac{1}{\sqrt{1+D/D_0}}. \tag{7}$$

where $D_0 = w_c f_2/f_1$ = size-independent constant called the transitional structure size. According to data fitting, D_0 can be considered as a constant, $D_0 \approx 200$ mm (7.9 in.). Factor θ represents for the ACI-446 size effect factor. Furthermore, in terms of fracture energy, the derivation yields $v_c = \sqrt{2EG_f / f_2w_c}$ = size-independent constant = average shear strength of concrete for small size beams. The value of v_0 is obtained by data fitting. Note that a basic assumption of the foregoing derivation is that, approximately, the cracks at maximum load must be geometrically similar. The recent tests of Syroka-Korol and Tejchman [22] give an excellent confirmation of such similarity; see Fig. **7.**

Eq. 7 represents an approximate size effect law, which has been experimentally verified for many kinds of brittle failures of reinforced concrete structures, as well as fiber composites, tough ceramics, rocks, foams, sea ice, wood, etc. In 2007, ACI-446 endorsed Eq. 7 for the shear capacity of concrete in failure of beams.

Note that when $D << D_0$, there is virtually no size effect. Hence, the case of normal size laboratory beams ($d \approx 0.25$ m) corresponds to plastic limit analysis, which implies no size effect. For very large beams (i.e. $d >> D_0$), Eq. 6 suggests that $\sigma_N \propto 1/\sqrt{D}$, Eq. 1, which gives a straight-line asymptote of slope −1/2 in the plot of log σ_N vs. log D. This asymptote, which gives the size effect of geometrically similar sharp cracks according to linear elastic fracture mechanics (LEFM), is generally beyond the range of practical applications.

Since the fracture energy is not relevant to the strength of very small beams (cca $D \leq$ 0.25 m), v_c should be evaluated by the best known method based on plastic limit analysis. The best choice of this method is not yet completely settled, but a point to note is that the size effect factor θ can be applied to the v_c value calculated according to any such method (e.g., those of Frosch, Hsu or MCFT). For example, according to the latest calibration in ACI-446, $V_0 = 12\, b_w c \sqrt{f'_c}$ where f'_c is in psi and V_0 in lb.; b_w = width of rectangular cross section or of the web. Based on the idea of Frosch and Wolf [28], c is the depth to neutral axis according to elastic analysis, i.e. $c = d[\sqrt{2\rho n + (\rho n)^2} - \rho n]$, which replaces d used previously to calculate the shear force in concrete. This replacement has the advantage that it takes approximately into account the effect of axial force or prestress force (the flanges on the sides of web must be ignored in calculating c [28]).

5 Meaningful and Misleading Statistical Evaluations from Large Database Encompassing Many Concretes and Labs

Committees of concrete societies nowadays try to compare various models for beam shear by statistics of prediction errors compared to a large database representing a collection of strength tests of beams made of all kinds of concretes, tested in different labs. Meaningful comparisons, however, are a complex problem with different overlapping trends and many random variables of widely different magnitudes of scatter, while the database is drawn from separate studies that had to be conducted without any coordinated scientific sampling scheme. In this kind of problem, the statistical inferences are tricky and can easily lead to misleading conclusions.

There has been a tendency to apply the statistics of data points to a problem that requires the statistics of trends, in our case the trend with respect to beam size (although the problem is the same for the trends with respect to shear span, steel ratio, etc.). What makes the comparisons particularly difficult is that (aside from relatively small experimental errors), random scatter types of very different magnitudes must be distinguished:

1) Scatter due to differences among concretes;
2) Scatter due to variation of beam size;
3) Scatter due to variations in shear span, steel ratio, concrete strength, aggregate size, type of prestress if any, rate of loading, concrete age, etc.

The scatter of type 1 happens to be an order of magnitude higher than the others and thus covers them up, including the statistical trend of the size effect factor. Furthermore, the fact the averages of *a/d*, ρ_w, etc. vary strongly through the subsequent size intervals, obfuscates the size effect trend. To make the cover-up conspicuous, Figs. **12** and **13** present an example comparing the latest joint database of ACI committtee 445D and Deutscher Ausschuss für Stahlbeton (DAfStb) with a size effect factor that is deliberately made to be absurdly erroneous, nonsensical. Then it is checked whether this error would be detected by the usual point-wise statistics. Fig. 13 compares two kinds of predictions:

1. Fig. 13 shows on top the plot of errors (or residuals) of the predictions according to the beam shear equation by Hsu, Yu, Le, Hubler, Cusatis and Bažant which uses the ACI-446 size effect factor θ, Eq. 7. The statistics is calculated by the method of ACI-445D (which uses uniform weights).

2. Fig. 13 (bottom) shows the plot of residuals calculated when the ACI- 446 size effect factor θ is deliberately perturbed as $\theta + \Delta\theta$, where $\Delta\theta$ is a nonsensical periodic perturbation expressed as

$$\Delta\theta = \frac{0.14}{\sqrt{1+d/d_0}}\cos[2\pi(\text{In}\, d \; - s)] \tag{8}$$

See the curves in Fig. **12**, in which d_0 = 254 mm (10 in.). Parameter s is a random phase shift which must be introduced to prevent bias due to the fact that the inflexion points of the cosine curve, which are placed arbitrarily, have no perturbation while all the others do. Phase shift s varies randomly with uniform probability between −0.5 and 0.5 (as obtained from a random number generator). These random shifts eliminate any evaluator's bias so that no point on the curve of size effect factor θ would get favored over the others. It may be noted, though, that if the shifts s were omitted, i.e., if s were fixed as 0, the resulting change in the coefficient of variation (C.o.V.) of errors would be about the same.

The question is whether the ACI-445D statistics can detect this deliberately nonsensical perturbation.

In the current ACI-445 statistics, the prediction is evaluated based on the ratio of test value V_{test} to the predicted one, V_{pre}. The errors (or residuals) of the predictions of data with the unperturbed and perturbed size effect factors are plotted as functions of ln$d$ in Fig. **13**, for all the data points in the ACI-445D database. The histograms of the errors are plotted on the right, again for both the unperturbed and perturbed cases. The perturbation is found to change the C.o.V. of the errors (root-mean-square error divided by the data mean) from 0.250 to 0.262, which is by less than 5% of the C.o.V.

It is now inescapable to note that both plots of the residuals, both histograms and both C.o.V.'s are almost the same. Evidently, the ACI-445D statistical method cannot distinguish a nonsensical model from a realistic one. How can it then be trusted for ranking various proposed models?

The assessment and ranking of various models clearly requires taking into account the scatters of type 2 and 3. The shape of the curve of log v_u versus log D must be checked by individual sets of data for the same concrete and the same lab before everything else. It is, likewise, important to check separately the trends of the type 3 effects.

Consequently, the proposed size effect factor must first be shown capable of fitting closely the individual test series on the same concrete and from the same lab. Subsequently, the proposed equation for beam shear strength needs to be compared to the individual tests and the C.o.V. computed for each. Then all these coefficients of variation need to be combined (in a root-mean-square manner) into one overall coefficient of variation or errors of the proposed beam shear equation. Alternatively, the overall coefficient of variation can be extracted from the multivariate optimization of the database fitted with an algorithm such as Levenberg-Marquardt's, in which all the parameters are varied simultaneously.

The fact that the available data cover only parts of size range makes it important that the size effect trend be supported by a sound theory and that the theory be validated by experiments. This is crucial for extrapolation of the size effect to large sizes. Currently, for beams without stirrups, 87% of the available tests pertain to $d <$ 508 mm (20 in.), 97% pertain to $d < 1.27$ m (50 in.), and none to more than 3048 mm (10 ft.), while, in practice, beams of depths exceeding 14.2 M (46 ft.) have been built.

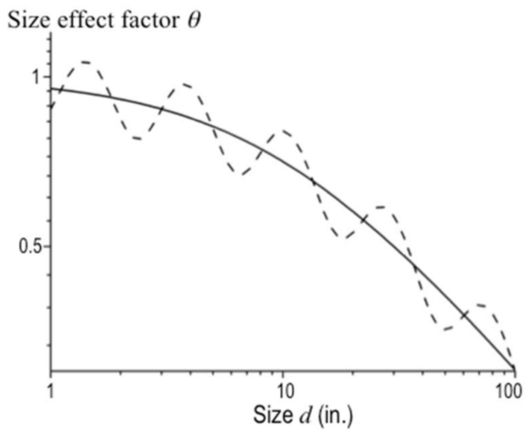

Fig. 12. Unperturbed size effect factor θ of ACI-446 (solid curve) and size effect factor perturbed by nonsensical cosinusoidal oscillation (dashed curve) (in the current ACI-318, the size effect factor is 1 for shear design).

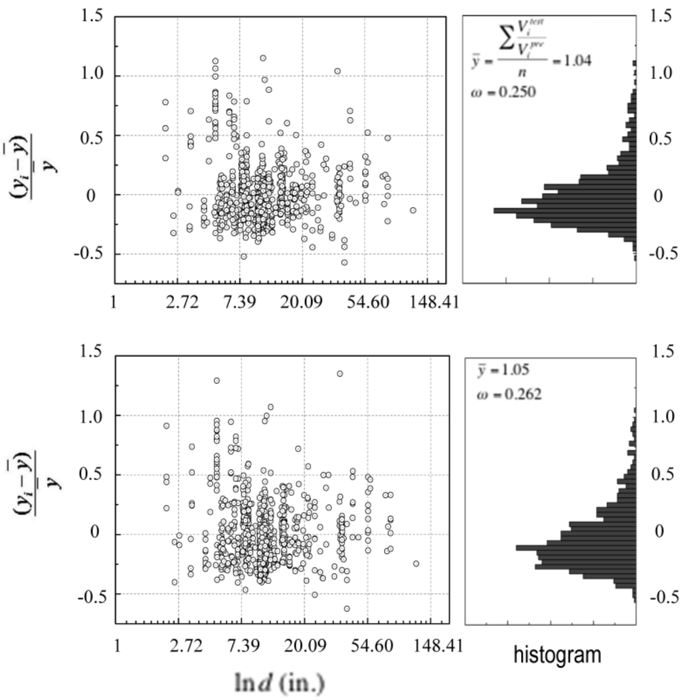

Fig. 13. Residuals (or errors) of unperturbed (top left) and perturbed (bottom left) ACI-446 size effect factor compared to all the data in the ACI-445D database (beams without stirrups), shown as a function of beam size, in semi-logarithmic scale. The column diagrams on the right are the histograms of the residuals (the size effect factor, which multiplies the concrete shear strength according to any model for small size beams, starts at small sizes with the value of 1; the oscillating perturbation shown is considered to have amplitude 0.14, which means the spread between maxima and minima is 0.28).

6 Conclusions and Closing Remarks

1. For small beam sizes, the plastic limit analysis is applicable and yields a small size estimate of the shear strength v_u due to concrete. The size effect factor θ should be applied to the v_u value obtained by the best plastic limit analysis (or strut-and-tie) model.
2. Unfortunately, the statistical evaluation of a beam shear formula must deal with a heteroscedastic database that has various kinds of strong bias. Mainly:

   a) the data are crowded for small sizes, very scant for large sizes and nonexistent for the largest sizes used in practice; b) The average shear span and reinforcement ratio of the tests in the database systematically decreases through subsequent size intervals. The bias must be filtered by statistical weights. The filtering demonstrates that, for large sizes,

   the size effect factor $\sigma_N \propto 1/\sqrt{d}$ (which represents the main resolution of ACI-446).
3. The JSCE Weibull type power law (adopted when no other size effect theory was known) is theoretically unjustified and does not agree with the subsequently accumulated data.
4. The Model Code size effect is an extreme case which is theoretically and experimentally unjustified, for several reasons: a) The increase of the shear force V_u with beam size terminates with a horizontal asymptote, which is an overlooked unphysical property, contradicted by tests; b) its derivation rests on invalid hypotheses based on the crack initiation state rather than the peak load state. The shear force is assumed to be transmitted by aggregate interlock distributed uniformly along uniformly

spaced diagonal cracks, rather than localizing into a single crack and also along the crack as the size increases. It is ignored that, in large beams, most of the shear force gets transmitted by an inclined compression strut above the main crack, in which the failure localizes as the size increases. The softening stress-separation law on the parallel cracks formed much below before the peak load is incorrect. It conflicts with the law known from fracture studies of concrete.

5. Dimensional analysis and the energy balance during crack formation inevitably lead to the size effect factor of ACI-446.
6. Evaluating different beam shear equations according to the coefficient of variation of the scatter of prediction errors (residuals) compared to the database points is fruitless and misleading. The reason is that the scatter due to differences among different concretes and testing labs, enhanced by the systematic variation of the average values of a/d, ρ_w in subsequent size intervals, obfuscates the trend of size effect. This cover- up is demonstrated by insensitivity to a large nonsensical sinusoidal perturbation of the size effect factor. If the statistical method cannot distinguish such nonsense, it cannot be used to compare different beam shear models.
7. It should also be noted that the deterministic size effect can, of course, be automatically taken into account when the structure is analyzed by finite elements based on a realistic constitutive model and damage-fracture concepts with a localization limiter (such a limiter is, for ex-ample, automatically featured in the widely used crack band model, which is utilized in software such as ATENA, SBETA, DIANA and is also

easily implemented in the user's material subroutine of other soft- ware such as UMAT or VUMAT of ABAQUS). The designs of large sensitive structures are increasingly subjected to checks by such finite element analysis. In that case, a high degree of safety is likely achieved even if the design code features a wrong size effect or none. Nevertheless, this is not yet a standard practice and, in that case, embedment of the size effect (of the correct form) in the design code is important.

8. Even if the design safety is checked by finite element analysis taking into account material uncertainty, embedment of the correct size effect in the design code is important for two other reasons: 1) to achieve an economic design, and 2) to allow a creative designer to exploit freely the true capacity of the material in daring new structural forms. As long as the size effect is incorrect, the code will, *a priori*, exclude some innovative, large and daring structural designs from consideration even if they are safe and pass the safety check by detailed finite element analysis.

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