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THE LATTICE DISCRETE PARTICLE MODEL (LDPM) FOR FRACTURE DYNAMICS AND RATE EFFECT IN CONCRETE: THEORY, CALIBRATION AND APPLICATIONS

BY

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22th DYMAT TECHNICAL MEETING

GRENOBLE
October 19-21th

Acknowledgements

Collaborators

- Daniele Pelessone & Xinwei Zhou (ES3)
- Jovanca Smith (University of West Indies)
- George Solomos (JRC)

Funding

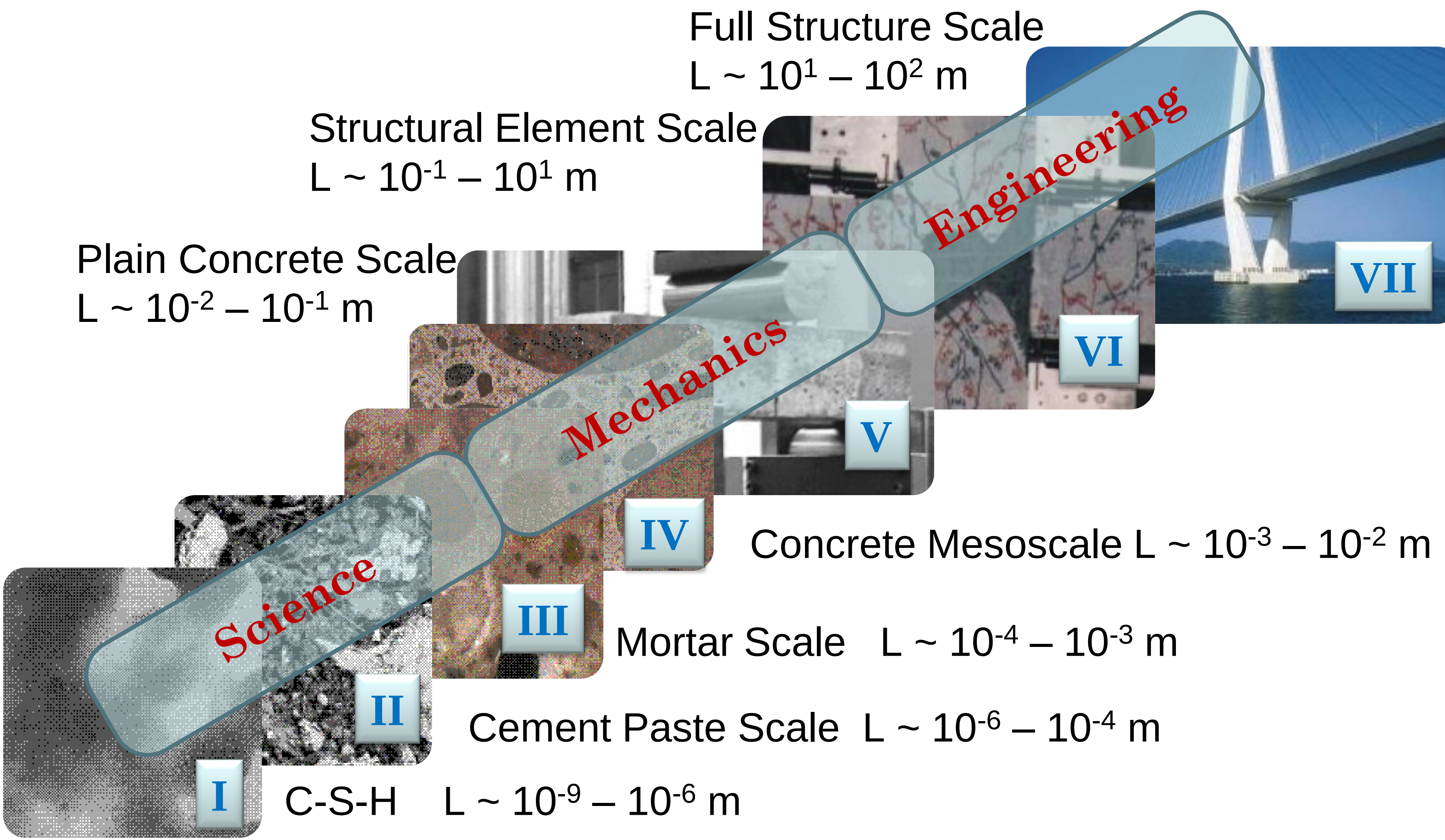
- Engineer Research Development Center
- National Science Foundation

Presentation Outline

- Introduction
- Lattice Discrete Particle Model (LDPM)
  - Calibration and Validation for Concrete and Fiber Reinforced Concrete
  - Simulation of Ultra High Performance Concrete
  - Dynamic Behavior and Rate-Effect
  - Projectile Penetration; Blast Analysis; Fragmentation
- MARS Software
- Conclusions

INTRODUCTION

The Multiple Length Scales of Concrete

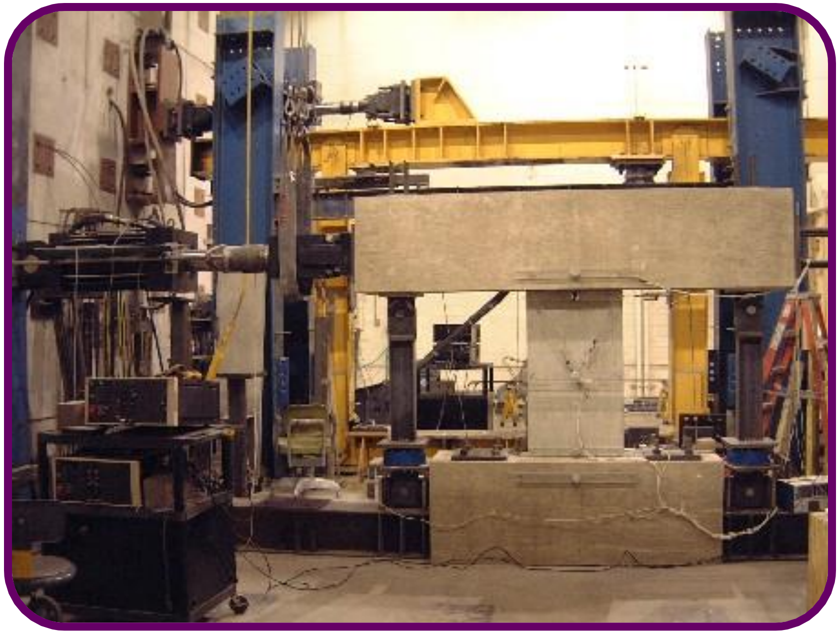


Where Should We Start?

At the Full Structure Scale?

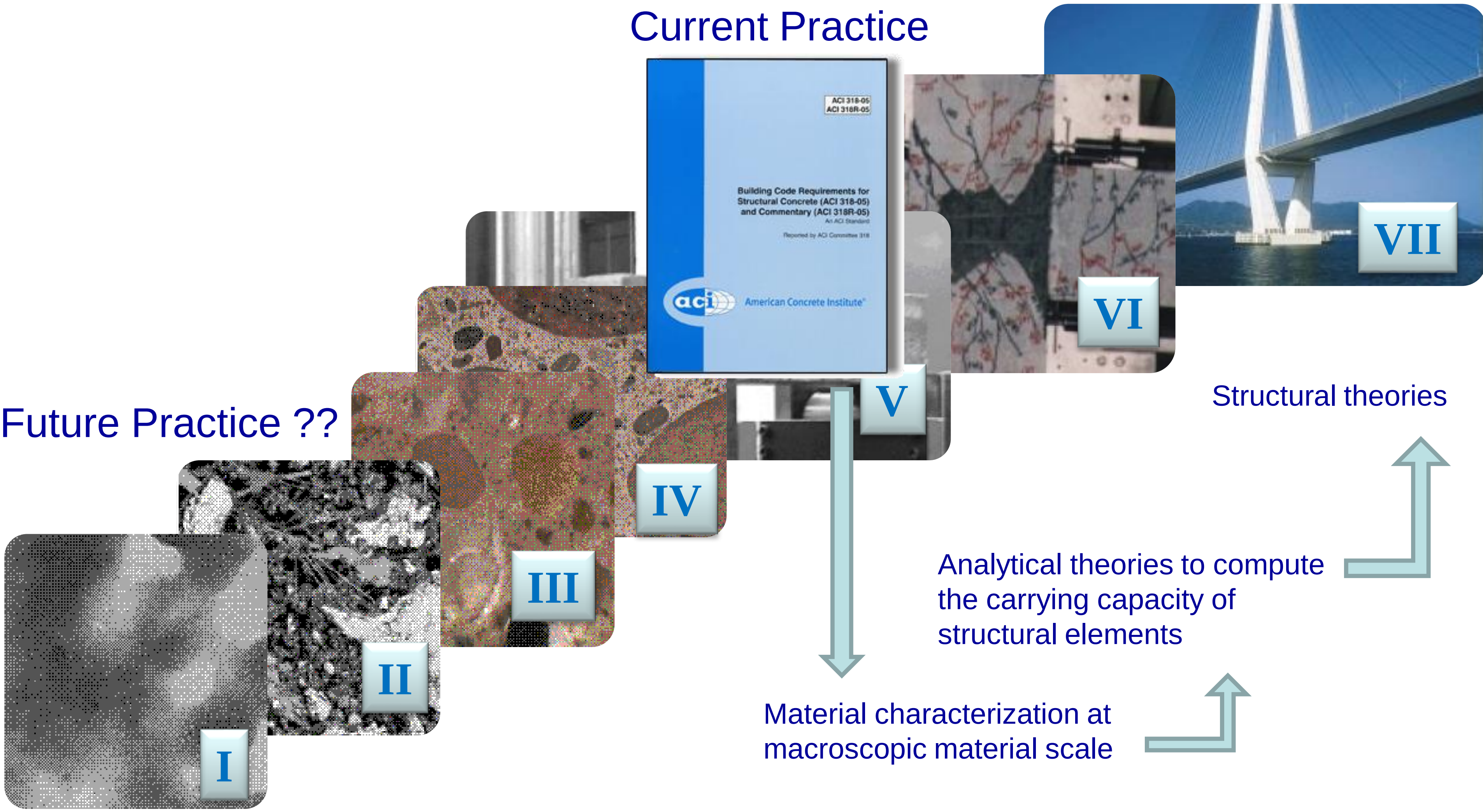
Duomo di Milano
Construction began 1386
Completion 1805
Not a good example of sustainable infrastructure!!!

At the Structural Element Scale?



Full scale tests are extremely expensive and time consuming
The time of “beam busting” must be over

Current Practice



Modeling at Different Length Scales

Full Structure Scale L ~ 10^1 – 10^2 m

- ✧ Structural theories
- ✧ FEM (Beams, Plates, Shells, …)
- ✧ ??

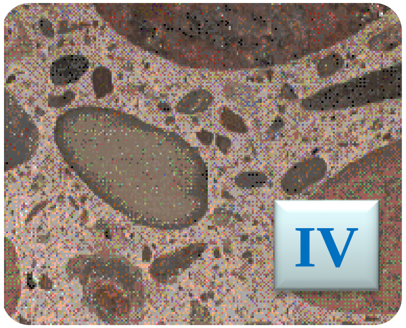
Structural Element Scale L ~ 10^{-1} – 10^1 m

- ✧ FEM (2D and 3D solid elements, Beam/Truss elements for reinforcement),

Plain Concrete Scale L ~ 10^{-2} – 10^{-1} m

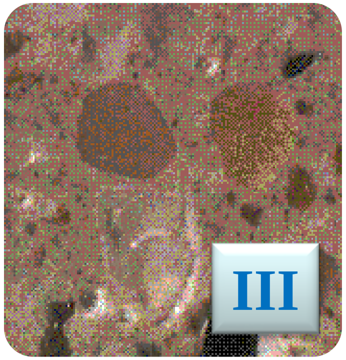
- ✧ Nonlinear fracture mechanics, Discrete modeling, Damage mechanics, Nonlocal theories, High-order theory, Peridynamics
- ✧ FEM, X-FEM, BEM, E-FEM, Meshless methods, Lattice/Particle models

Modeling at Different Length Scales, Cont.



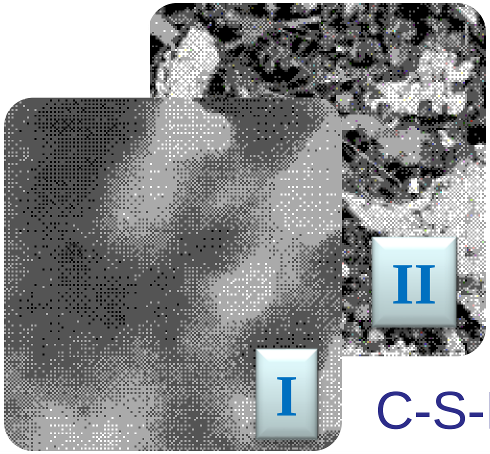
Concrete Mesoscale L ~ 10^{-3} – 10^{-2} m

- ✧ **Lattice Discrete Particle Model(LDPM)**, Lattice models, DEM



Mortar Scale L ~ 10^{-4} – 10^{-3} m

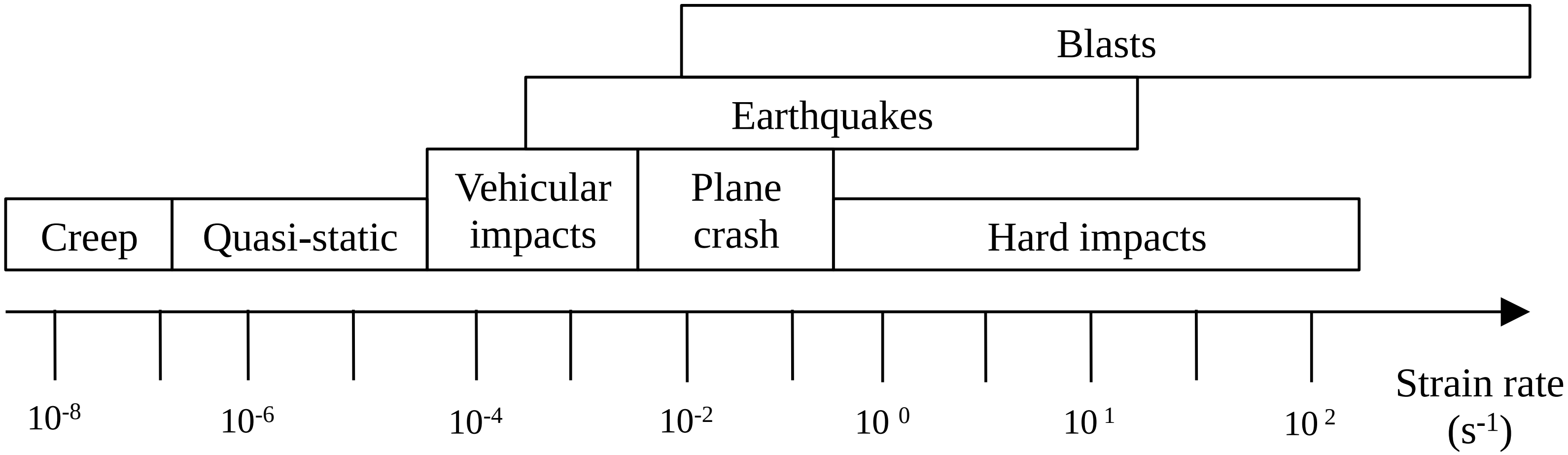
- ✧ FE Numerical Concrete, RBSN



Cement Paste Scale L ~ 10^{-6} – 10^{-4} m

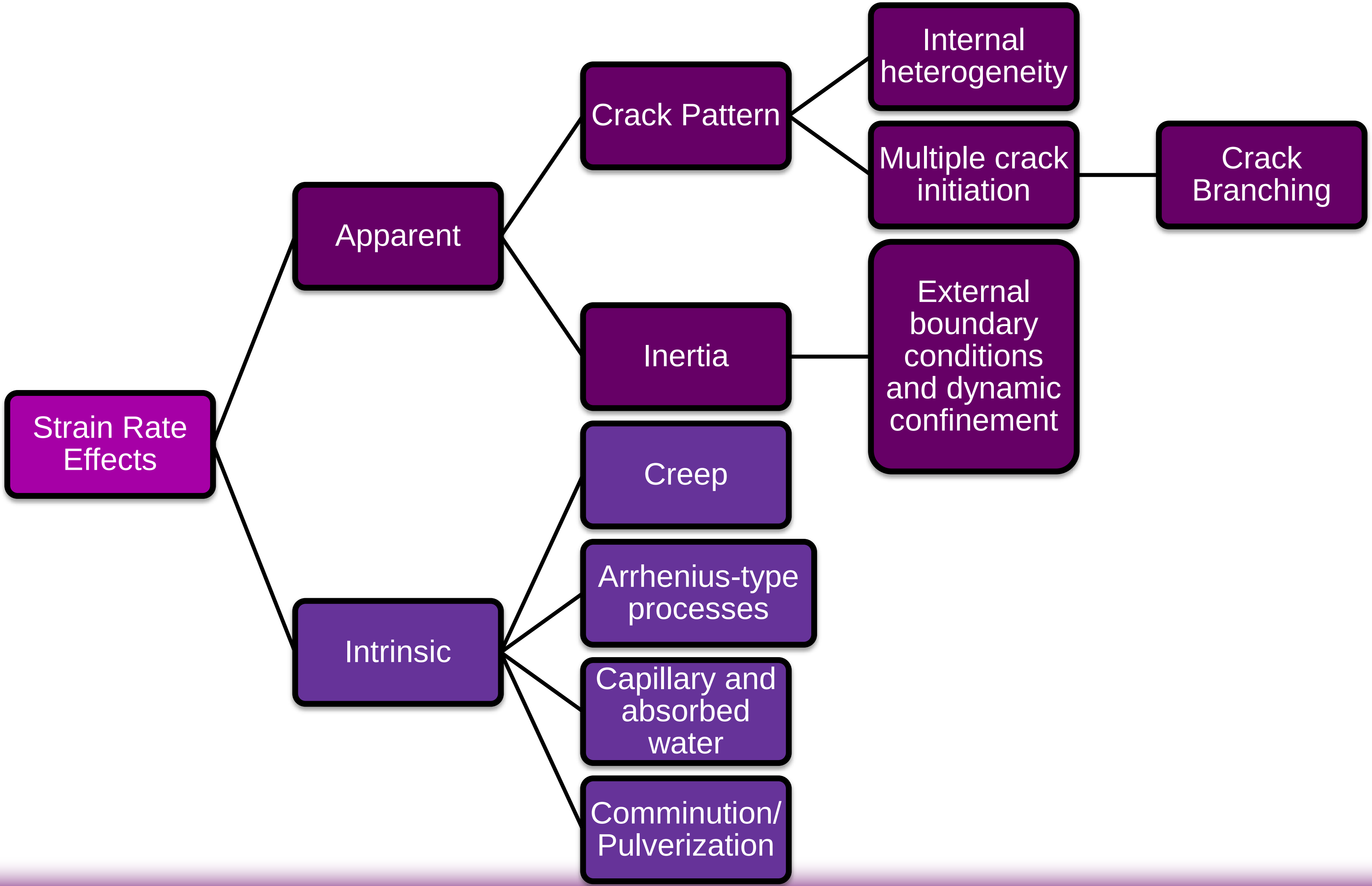
C-S-H L ~ 10^{-9} – 10^{-6} m / MD/Atomistic Simulations

Strain Rate Dependence of Concrete



Typical strain rates for various types of loading (Bischoff and Perry, 1991)

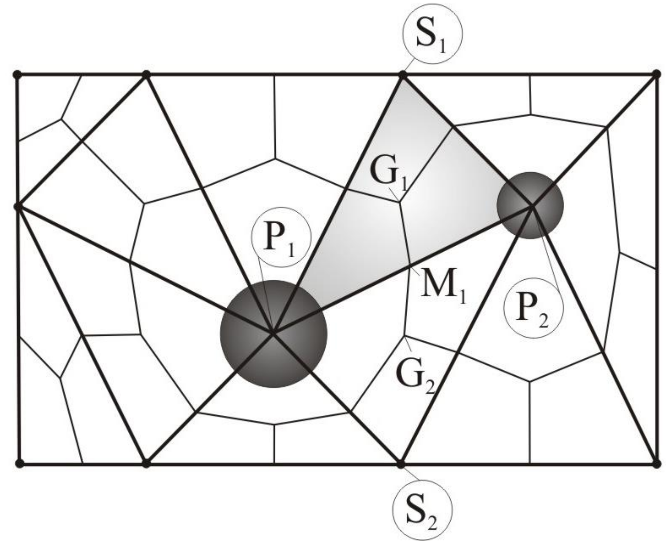
Strain Rate Dependence of Concrete



LATTICE DISCRETE PARTICLE MODELING OF CONCRETE

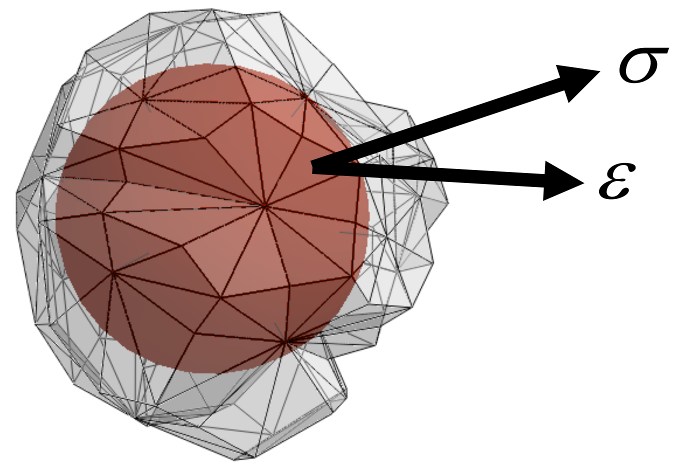
Lattice Discrete Particle Model (LDPM)

- *A priori volume discretization* is performed taking into account *material heterogeneity (coarse aggregate pieces)*



- *Delaunay triangulation* provides volume subdivision into tetrahedra starting from aggregate centers

- *A dual tessellation* of the triangulated domain defines a set of *discrete polyhedral cells*



- The external triangular faces are the *facets* through which adjacent cells interact

Lattice Discrete Particle Model (LDPM)

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- Stresses and strains vectors are defined on *tessellation facets.* Stresses and strains are defined on a *discrete number of orientations*

- *Discrete compatibility equations* (strains vs. displacements) are formulated through the relative displacements (and rotations) of adjacent nodes (particles)

- *Discrete equilibrium equations* are obtained through the equilibrium of each discrete cell

- Vectorial constitutive equations

    - Softening behavior is only associated with tensile stresses (fracture)

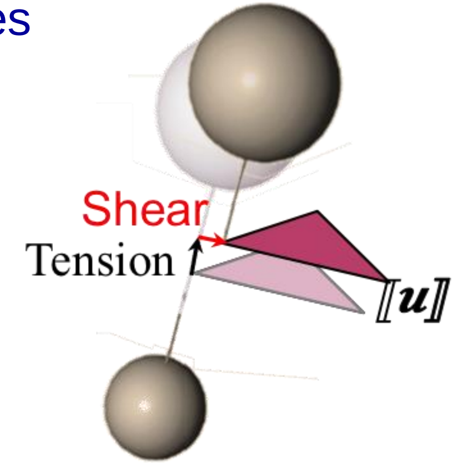
    - Compressive behavior is always hardening (compaction)

    - Shear behavior simulates cohesion and friction

LDPM Vectorial Constitutive Law

- Discrete compatibility equations (strains vs. displacements) are formulated through the relative displacements (rotations included) of adjacent nodes

$$\varepsilon_N = \frac{\boldsymbol{n}^T[\![\boldsymbol{u}]\!]}{L} \qquad \varepsilon_M = \frac{\boldsymbol{m}^T[\![\boldsymbol{u}]\!]}{L} \qquad \varepsilon_L = \frac{\boldsymbol{l}^T[\![\boldsymbol{u}]\!]}{L}$$



- Fracturing Behavior - $\varepsilon_N > 0$

Shear strain

$$\varepsilon_T = \sqrt{\varepsilon_M^2 + \varepsilon_L^2}$$

Coupling strain

$$\tan\omega = \frac{\varepsilon_N}{\sqrt{\alpha}\,\varepsilon_T}$$

Equivalent strain

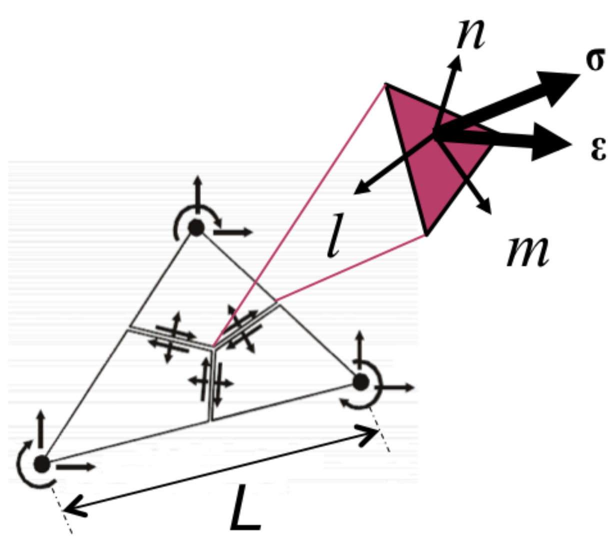
$$\varepsilon = \sqrt{\varepsilon_N^2 + \varepsilon_T^2}$$

Equivalent stress

$$\dot{\sigma} = E\dot{\varepsilon} \qquad 0 \le \sigma \le \sigma_b(\varepsilon,\omega)$$

where

$$\sigma_b(\varepsilon,\omega) = \sigma_0(\omega)\exp\left\{K(\omega)\frac{\langle \varepsilon - \varepsilon_0(\omega)\rangle}{\sigma_0(\omega)}\right\}$$



Normal stress

$$\sigma_N = \frac{\sigma}{\varepsilon}\varepsilon_N$$

Tangential stress

$$\sigma_T = \frac{\sigma}{\varepsilon}\varepsilon_T$$

Coupling stress

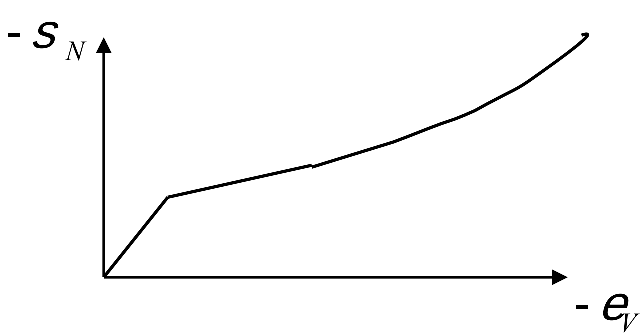
$$\tan\omega = \frac{\sigma_N}{\sigma_T/\sqrt{\alpha}}$$

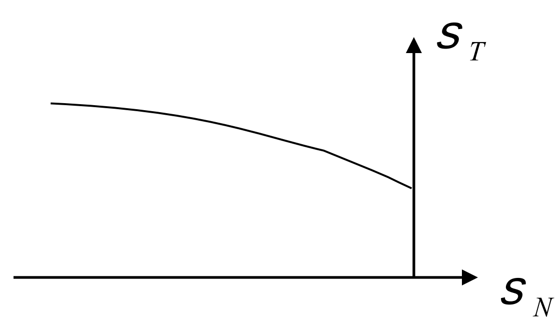
LDPM Vectorial Constitutive Law

- Frictional Behavior/Compaction - $\varepsilon_N < 0$

Normal stress
$$\sigma_N = \begin{cases} F_N(\varepsilon_V) = -\sigma_c + K_c(\varepsilon_V + \varepsilon_C), & -\varepsilon_V < 0 \\ F_N(\varepsilon_V) = -\sigma_{c0}\exp\left(-K_c \dfrac{\varepsilon_V + \varepsilon_{c0}}{\sigma_{c0}}\right) & -\varepsilon_V \geq \varepsilon_{c0} \end{cases}$$

Shear stress
$$\sigma_T = F_T(\sigma_N) = \sigma_S + (\mu_0 - \mu_\infty)\sigma_{N0} - \mu_\infty \sigma_N - (\mu_0 - \mu_\infty)\sigma_{N0}\exp\left(\sigma_N / \sigma_{N0}\right)$$





- Strain Rate Dependence -

Tension stress-strain boundary and the cohesion is scaled by a function of the strain rate, $\dot{\varepsilon}$:

$$F(\dot{\varepsilon}) = 1 + c_1 \operatorname{asinh}\left(\dot{\varepsilon} / c_2\right)$$

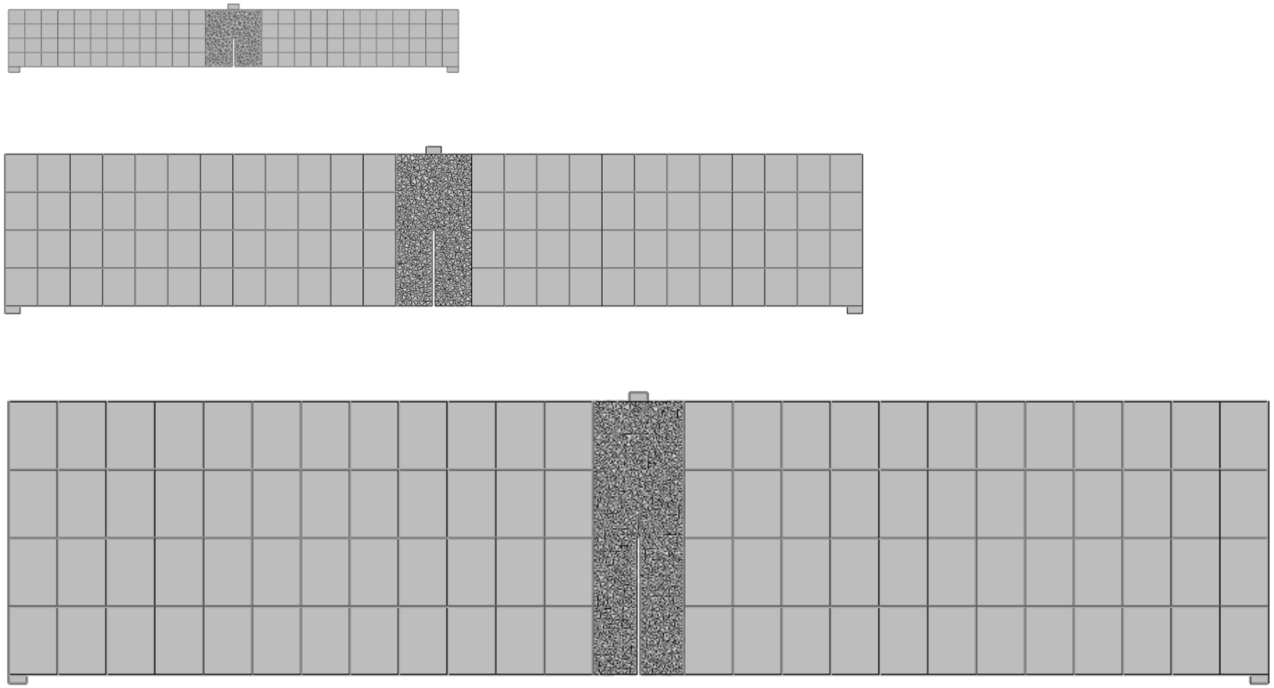
LDPM Modeling Capabilities

- Uniaxial compression tests
- Biaxial compression tests
- Triaxial compression tests with reverse of softening into hardening
- Hydrostatic and Uniaxial Strain compression tests
- Direct tensile tests; Brazilian tests
- Module of rupture
- Mode I and Mixed mode fracture tests
- Energetic size effect
- Cycling loading
- Anchor extraction
- Projectile penetration
- Blast induced fragmentation
- Impact induced fragmentation
- ASR deterioration
- Coupling with heat transfer and multiple species transport

Calibration and Validation

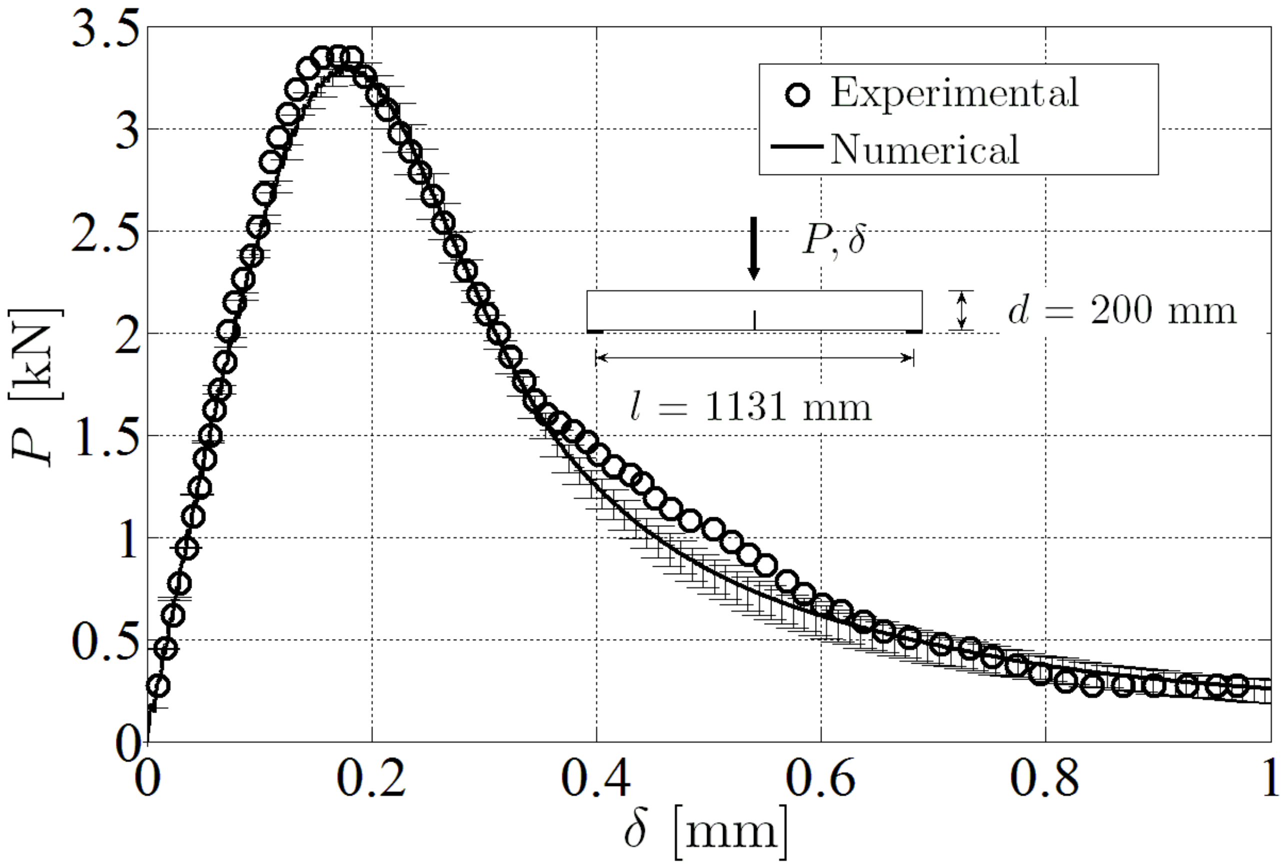
- The calibration of the model requires (at least) the following set of data: 1) Uniaxial Compression Tests, 2) Hydrostatic Compression Tests, 3) Fracture Tests
- These data must be either obtained through direct experimentation or estimated from published experimental data
- Validation is performed by simulating additional experimental data without further adjustment of model parameters

Example: Fracture Tests



- Fracture specimens (Medium (*D* =200 mm) used for calibration, Small (*D* = 100 mm) and Large (*D* = 300 mm) used for validation)

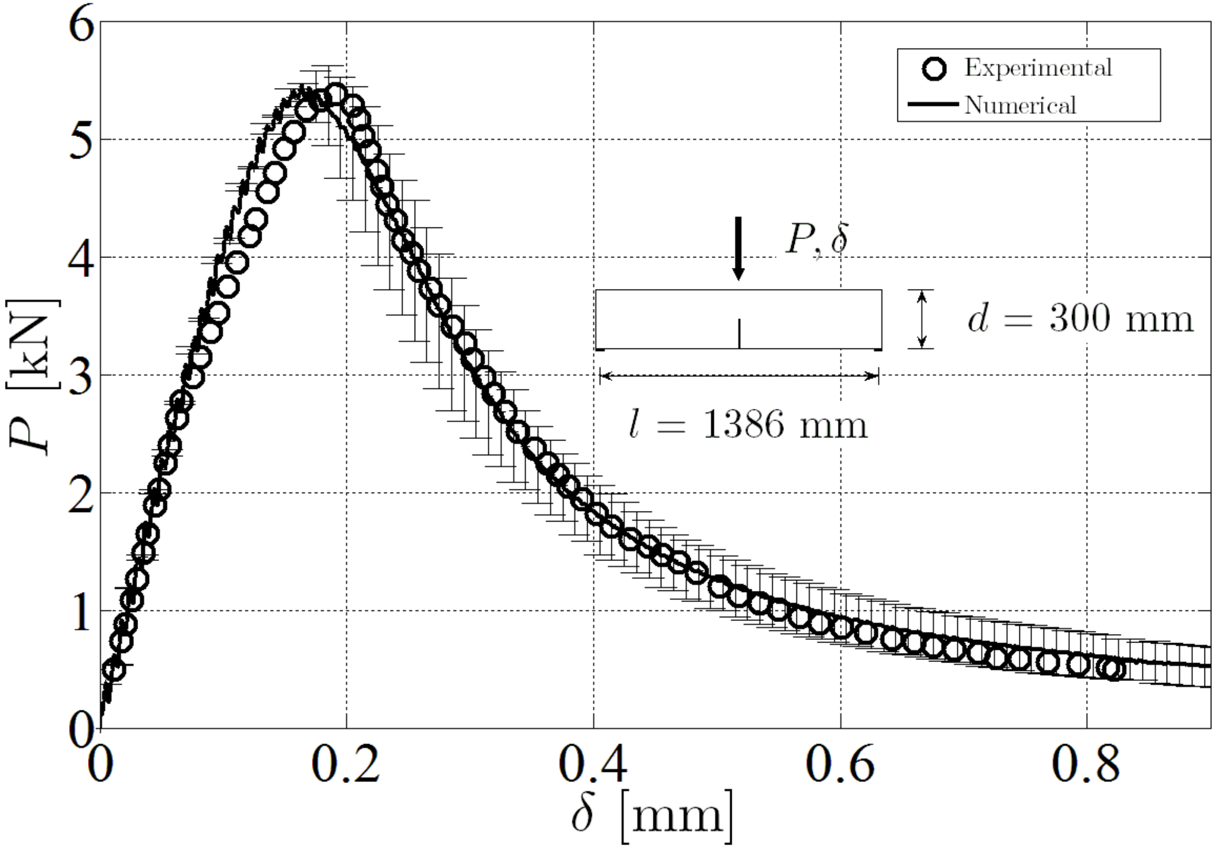
Fracture Tests : Calibration



Three-point bending test on the medium-size specimen (plus unconfined uniaxial compression test and hydrostatic test – not shown)

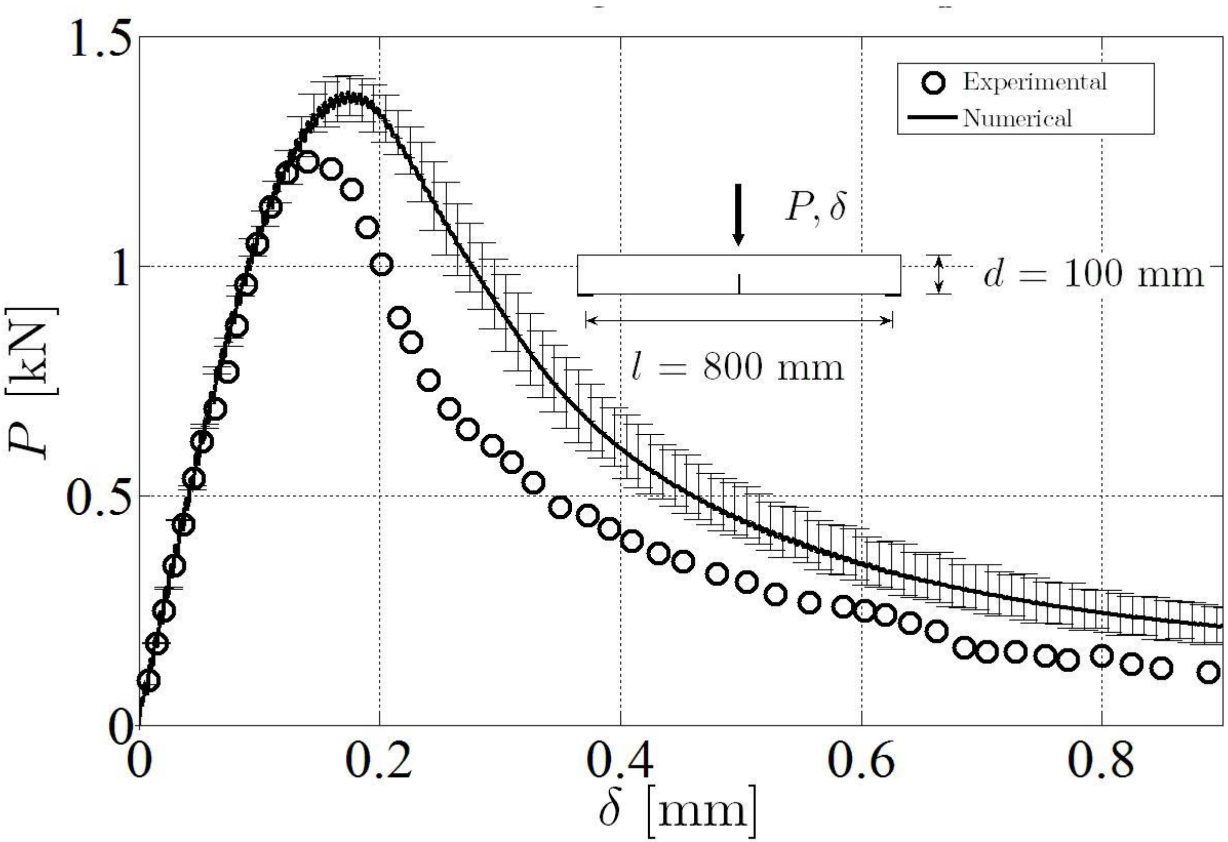
Fracture Tests: Validation

Three-point bending test of the large-size specimen



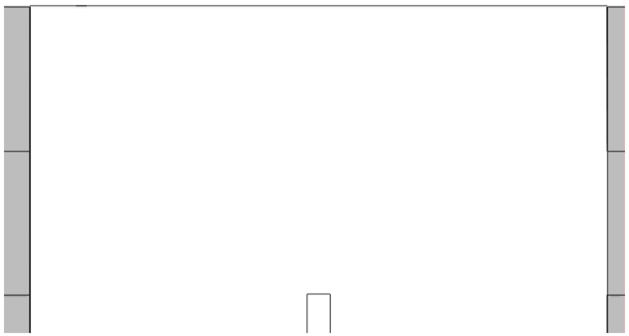
Fracture Tests: Validation, Cont.

Three-point bending test of the small-size specimen

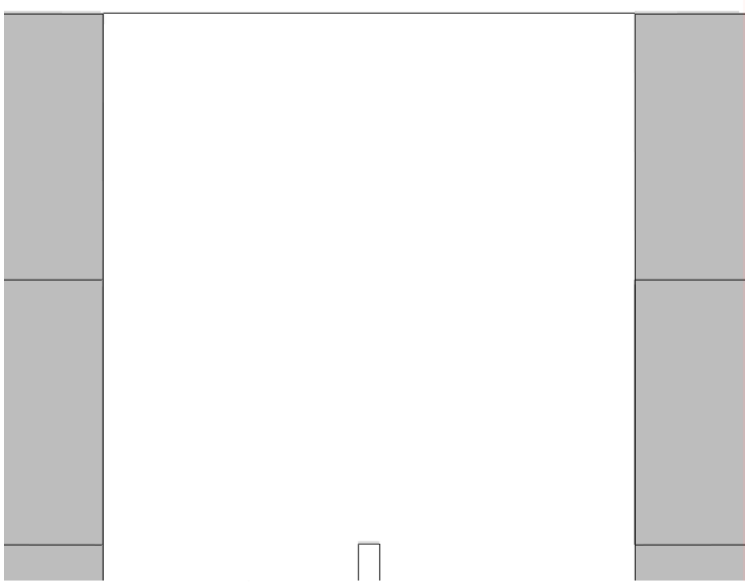


Fracture Tests: Animations

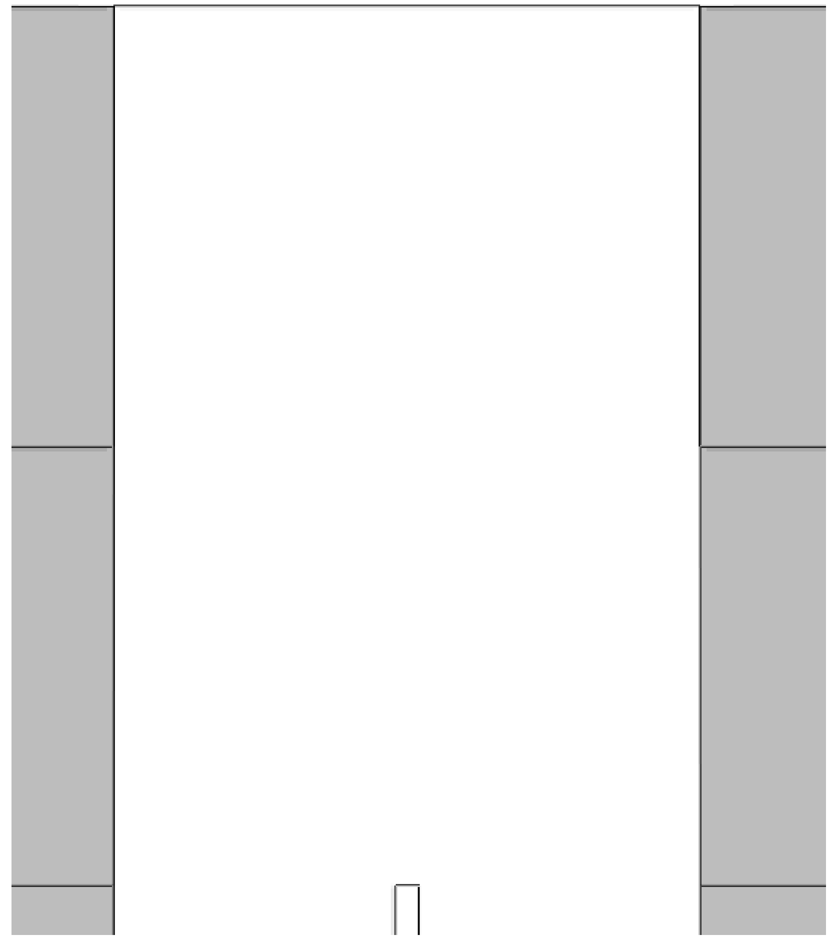
SMALL



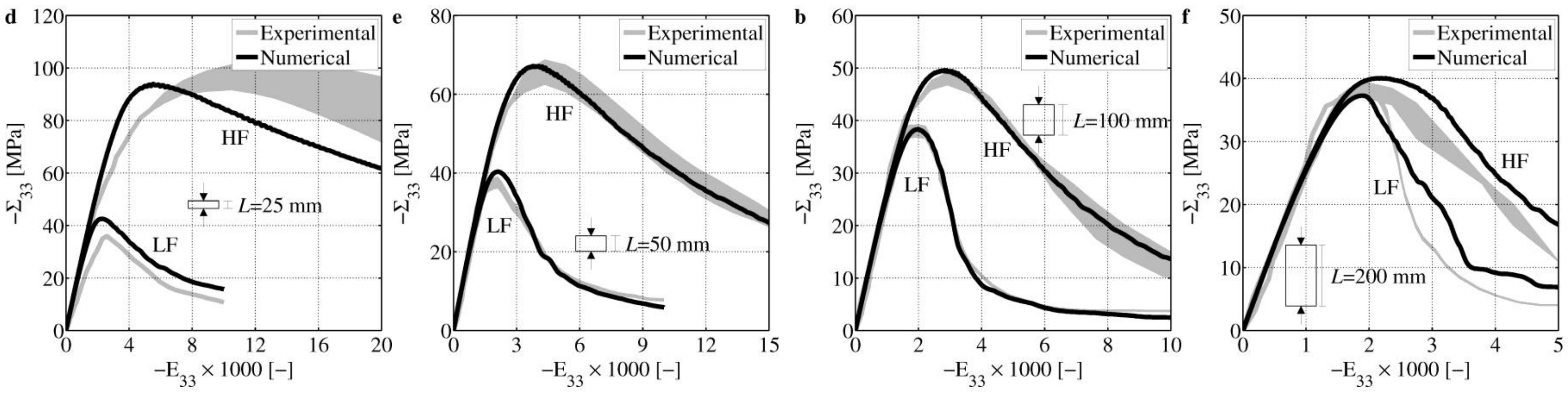
MEDIUM



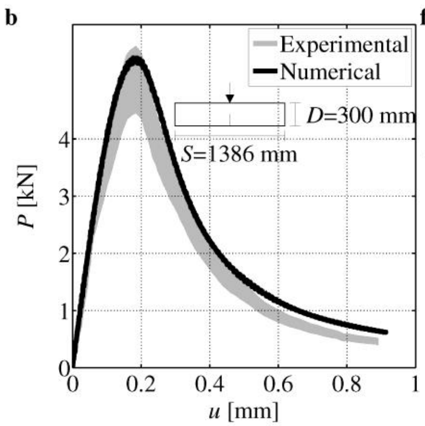
LARGE



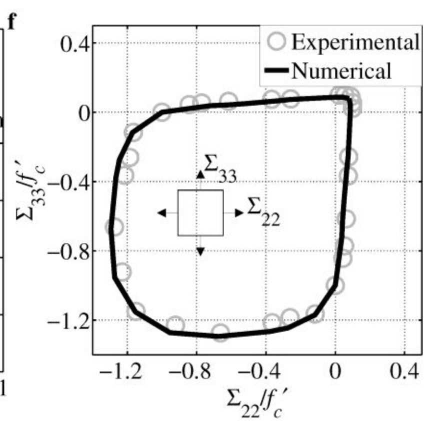
LDPM Modeling Capabilities



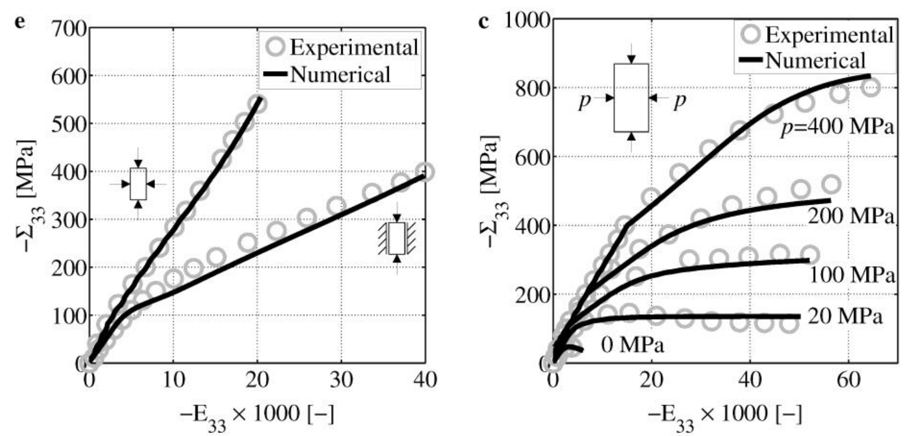
Unconfined Compression



Tensile Fracture

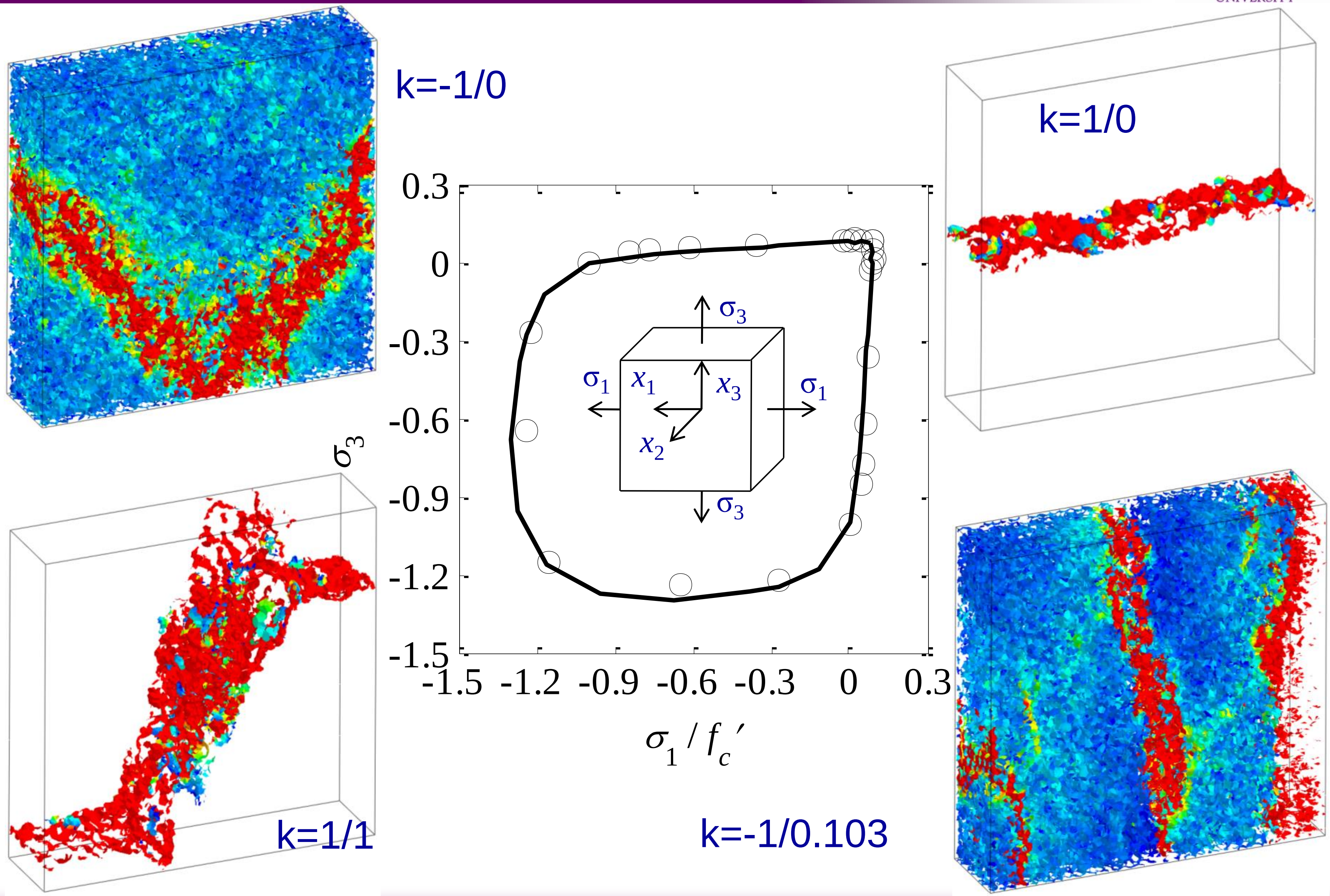


Biaxial Compression



Triaxial Compression

Biaxial Behavior: Failure Modes

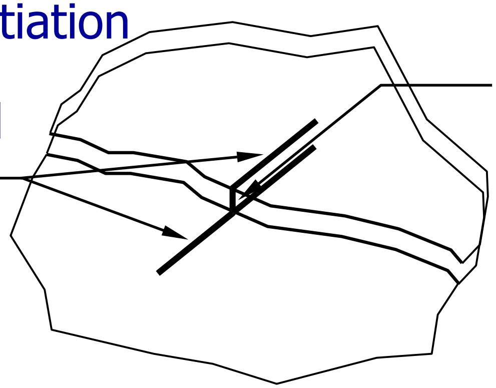


Fiber Addition (LDPM-F)

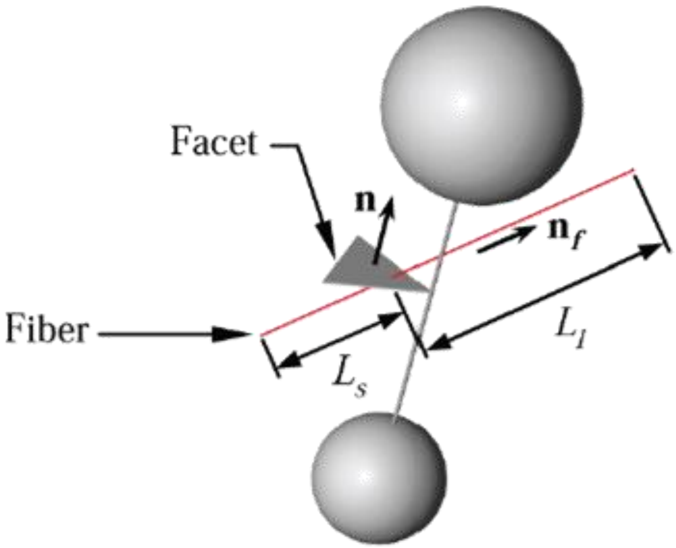
Reinforcing fiber at crack initiation

Two embedded segments

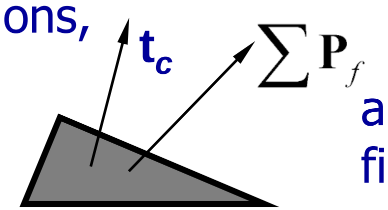
One "bridging" segment across crack



Reinforcing fiber as crack width increases in random direction

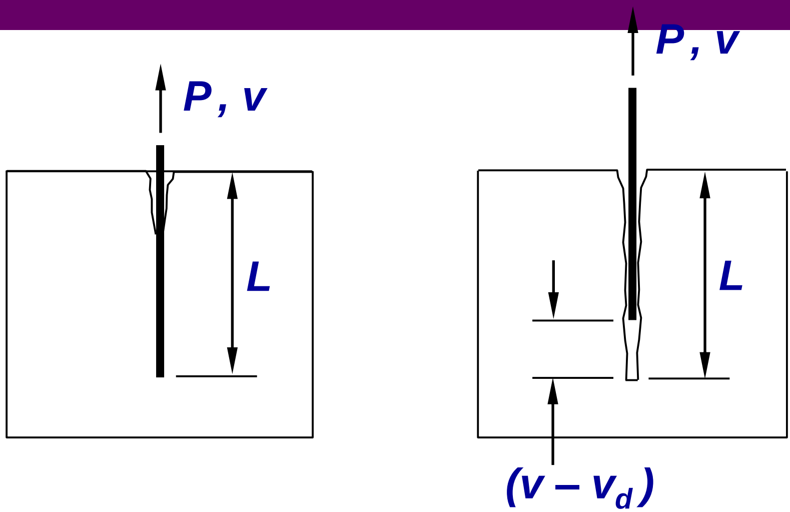


LDPM triangular facets (cracks) subject both concrete tractions,



and forces from fiber bridging segments.

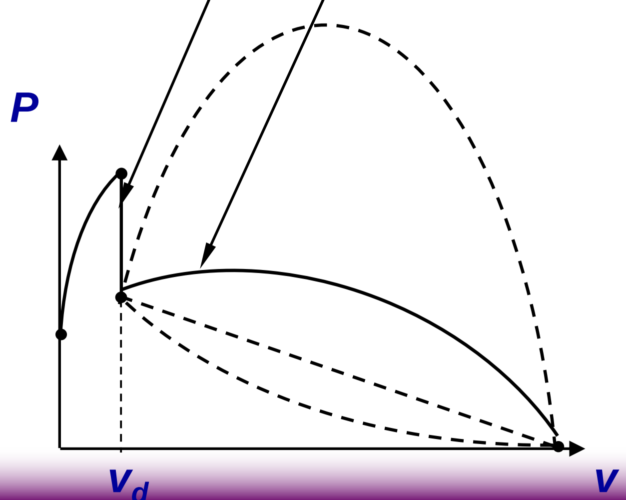
Fiber-Concrete Interaction



- P is the force, v is the fiber displacement, L is embedment length
- A constant friction stress and a debonding fracture energy affects the initial resistance of the fiber to separate from the concrete.

- After debonding:

1) sudden load drop as resistance shifts to a purely frictional nature
2) frictional pullout characterized by slip-hardening coefficient, β

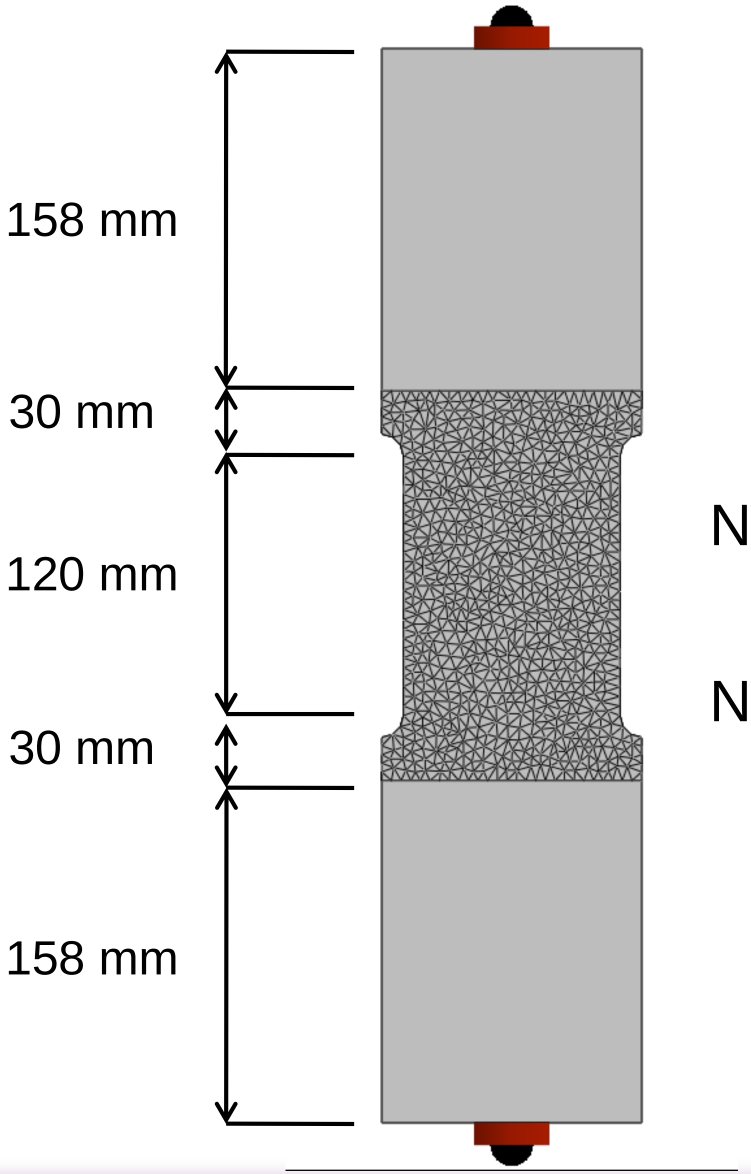


$\beta < 0$: slip-softening;

$\beta > 0$: slip-hardening; possibility of fiber rupture

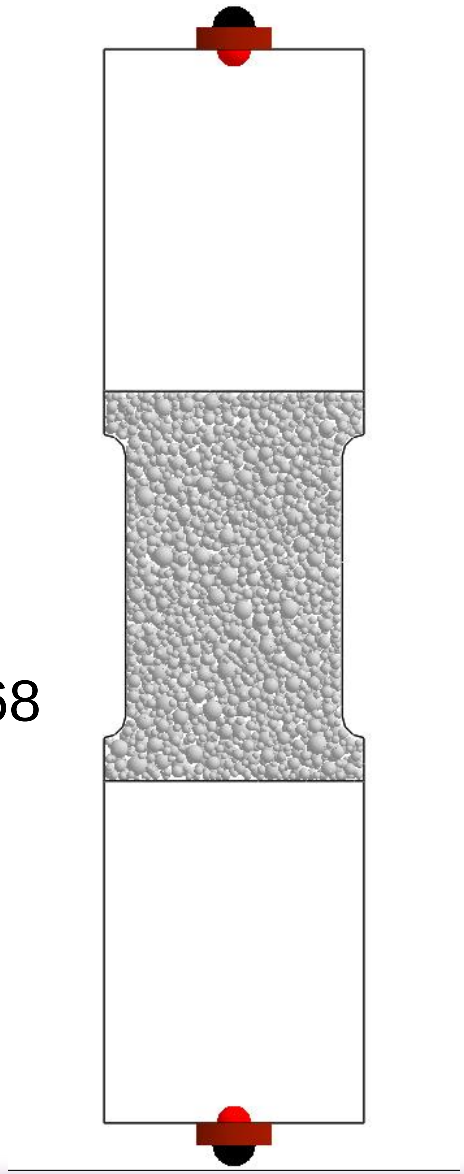
$\beta = 0$: interface friction independent of slip

FRC Specimen Geometry

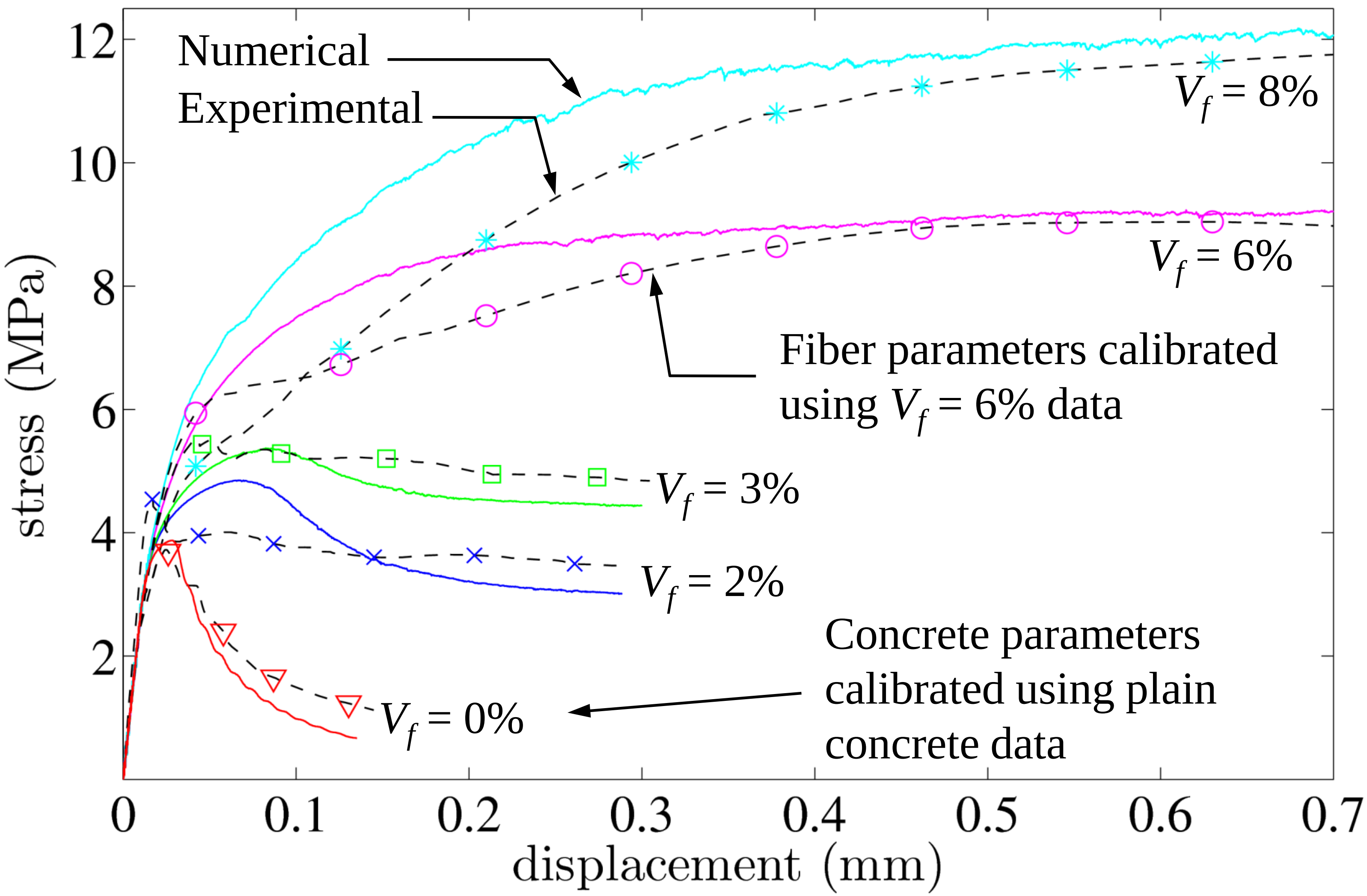


No. tets: 4233

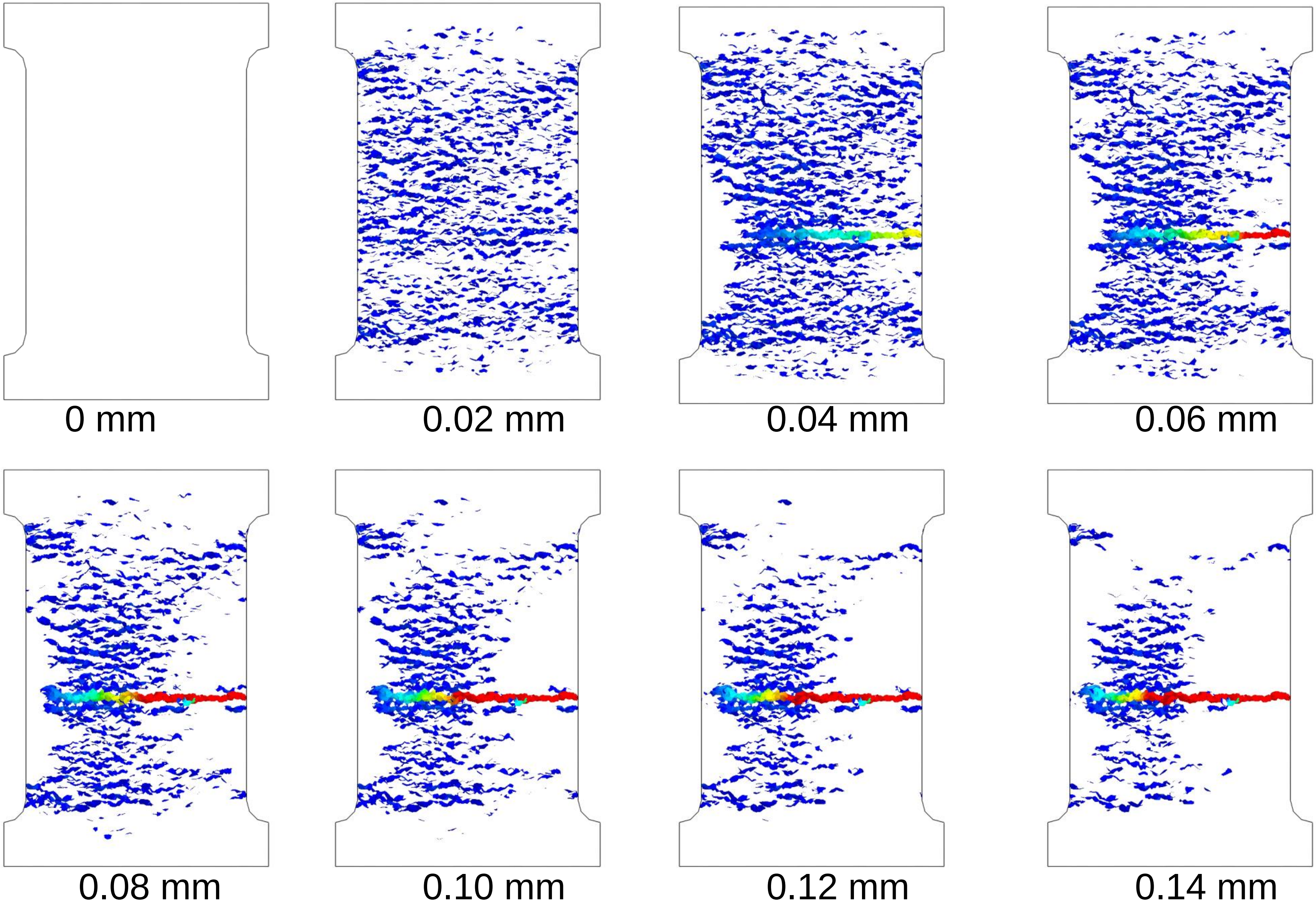
No. particles: 19668



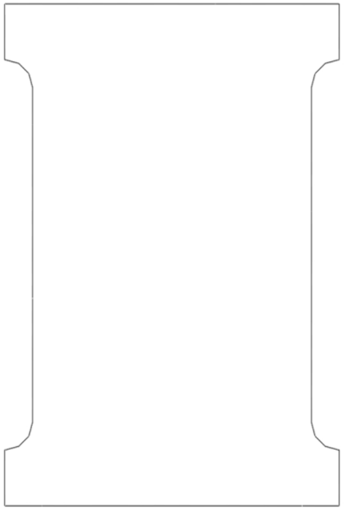
Stress vs. Disp. Curves, Steel Fibers



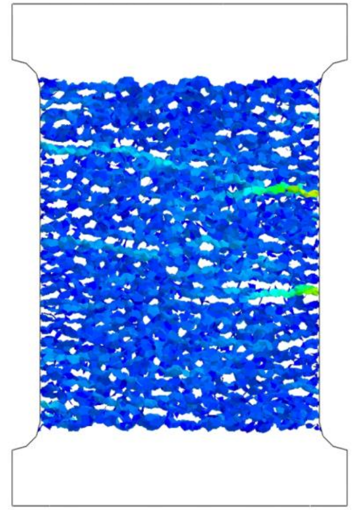
Crack Distribution for $V_f = 0\%$

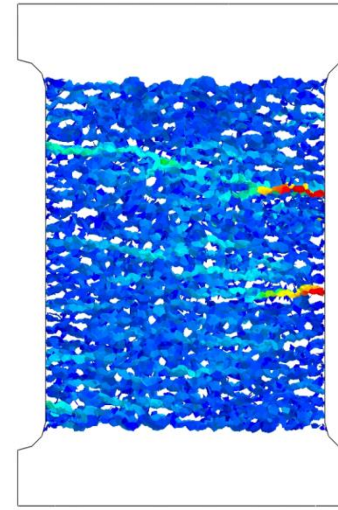


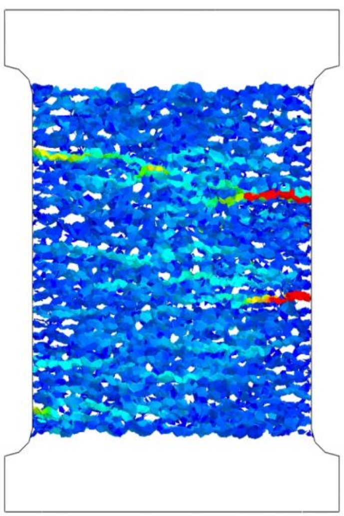
Crack Distribution for $V_f = 6\%$

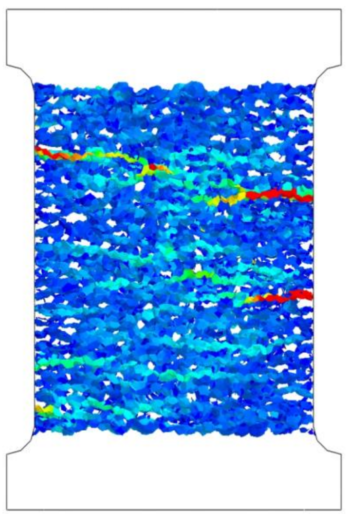

0 mm

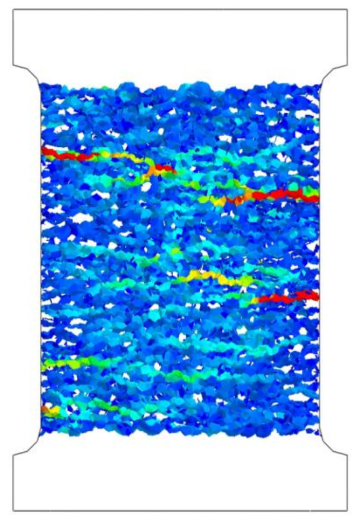

0.06 mm

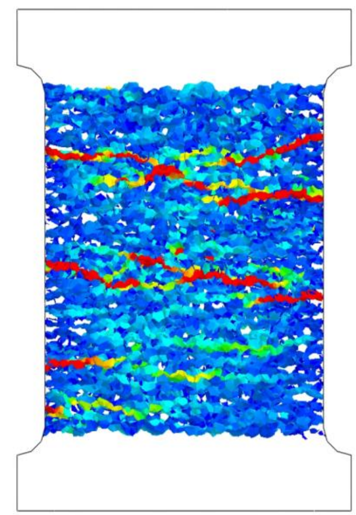

0.12 mm


0.18 mm

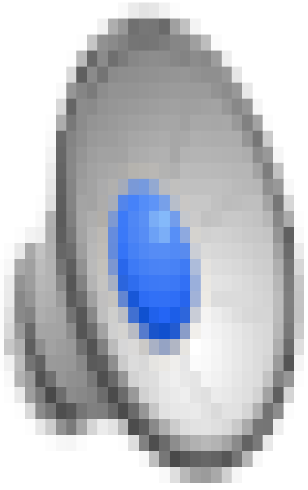

0.24 mm

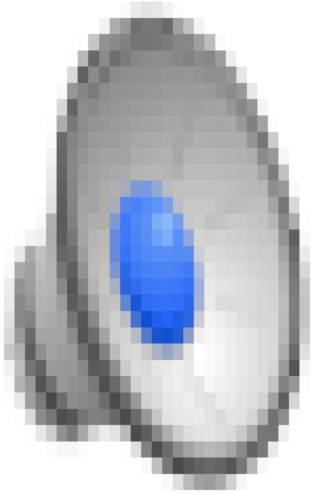

0.30 mm


0.36 mm


0.60 mm

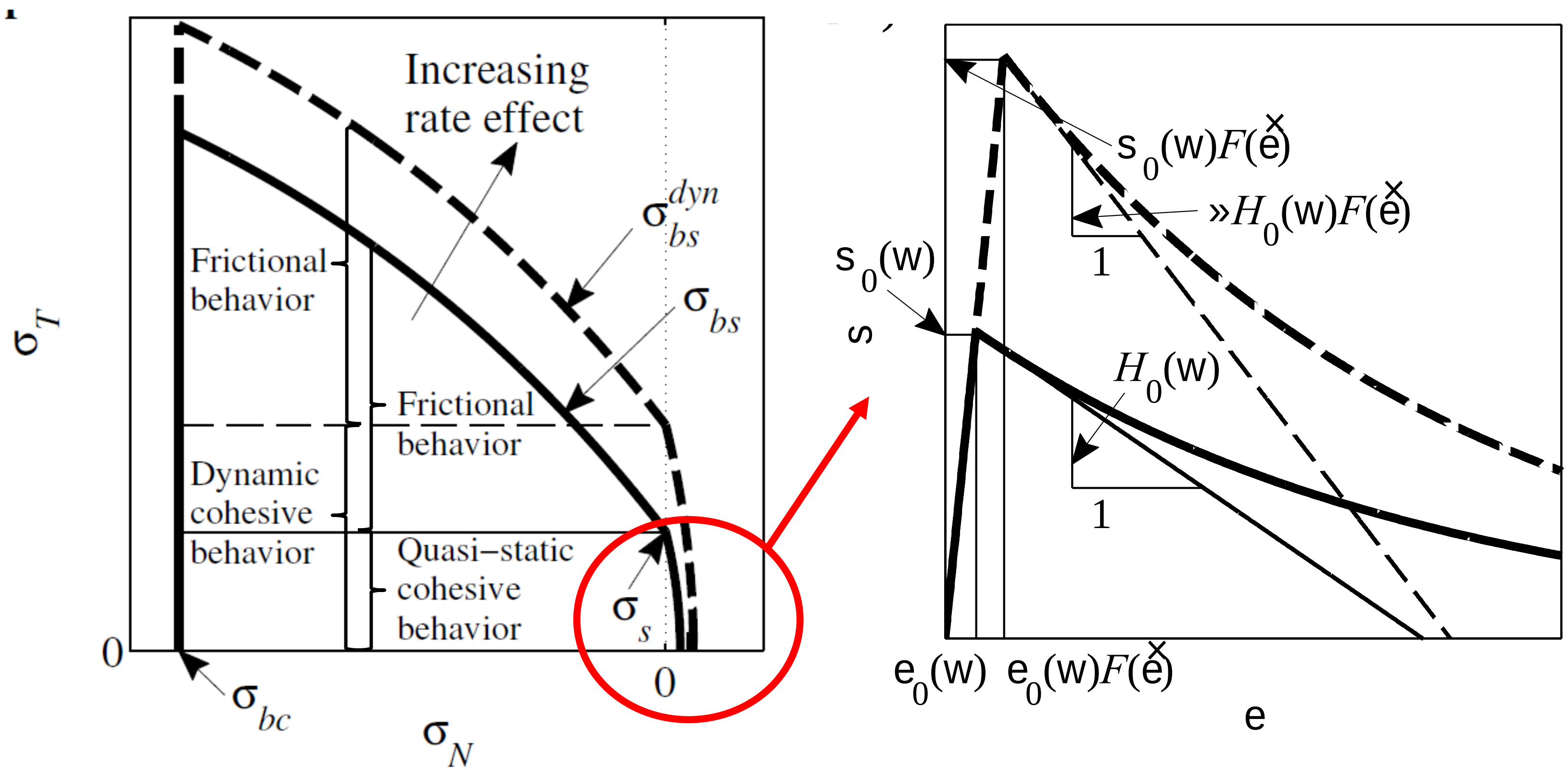
Animation for V_f = 0 and 6%



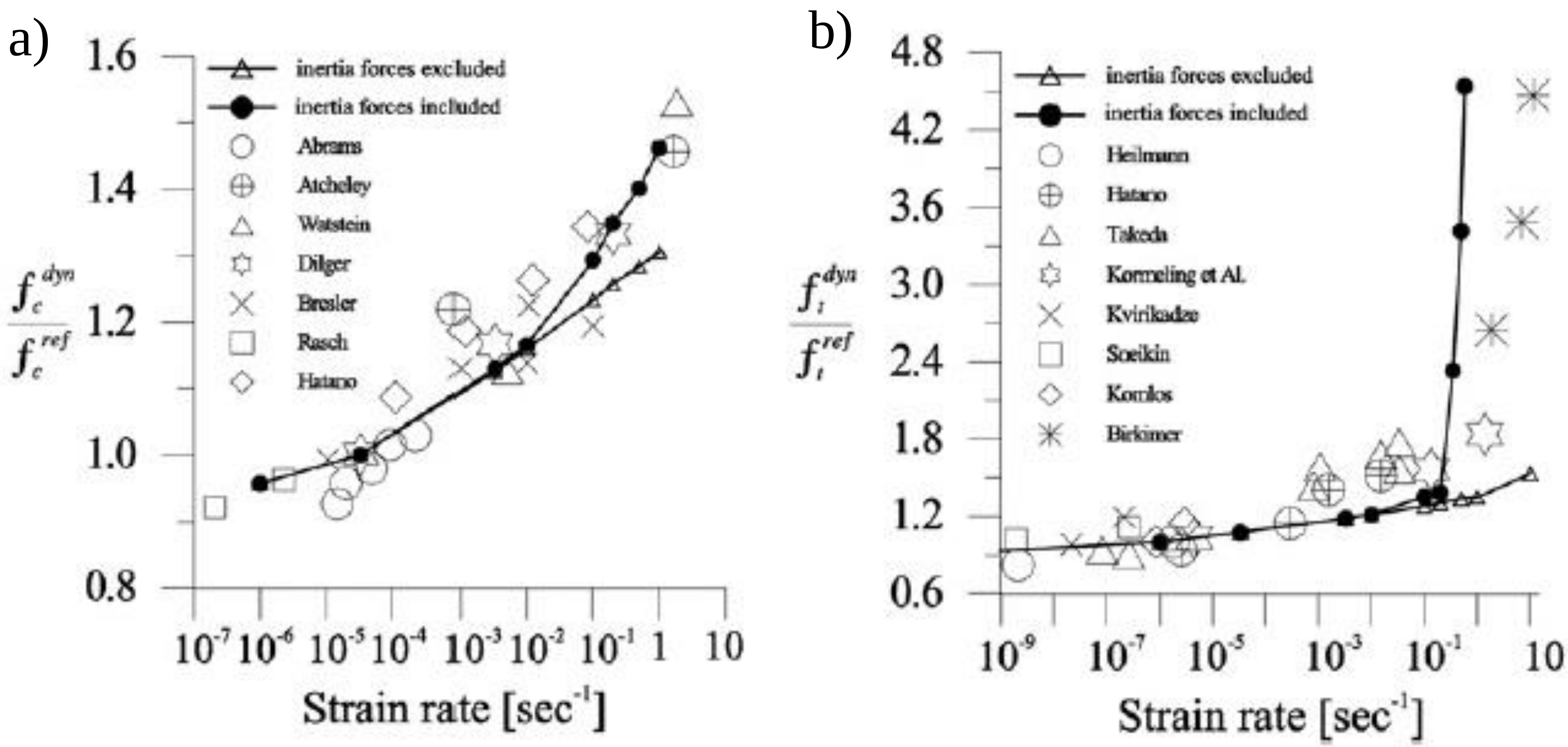


Strain Rate Dependent Formulation

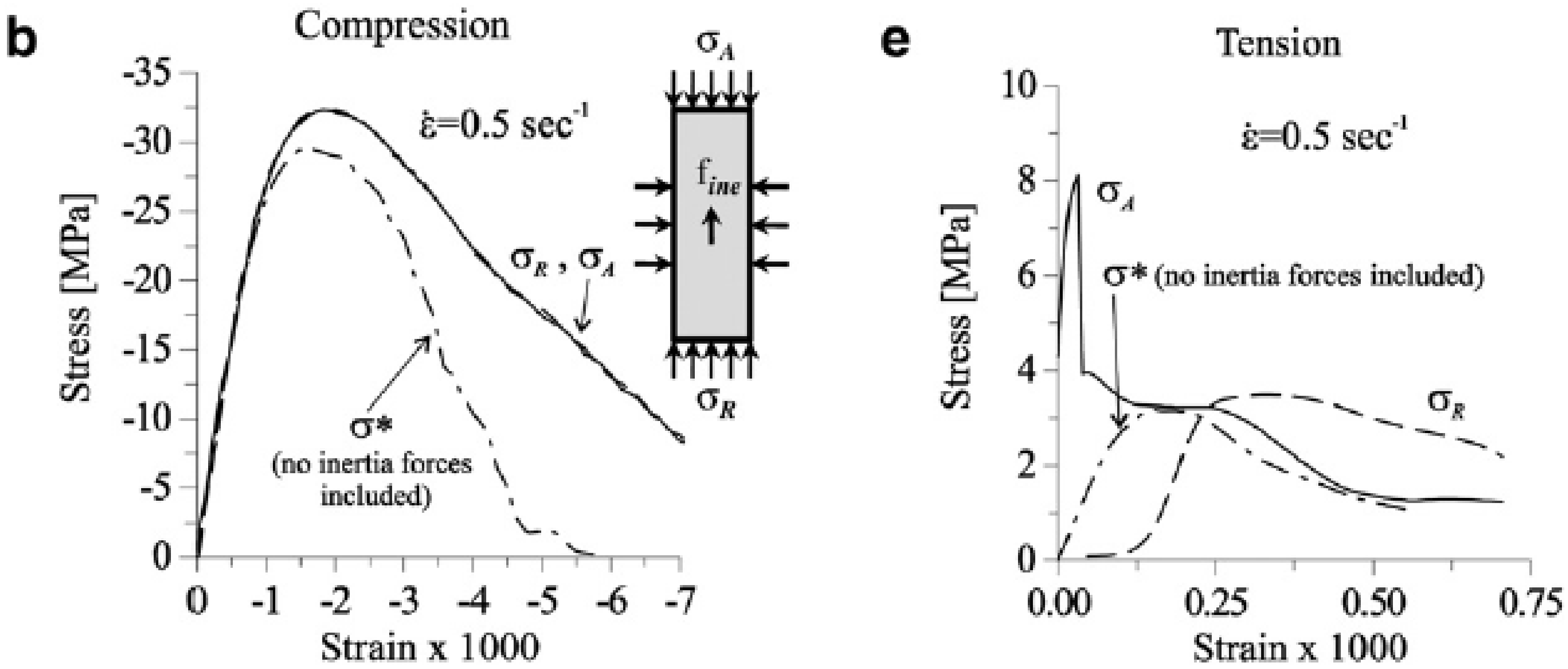
$$F(\dot{e}) = \left[1 + c_1 a \sinh\left(\frac{\dot{e}}{c_0/l} \right) \right]$$



Rate Effect and Dynamic Increase Factor



Effect of Inertia



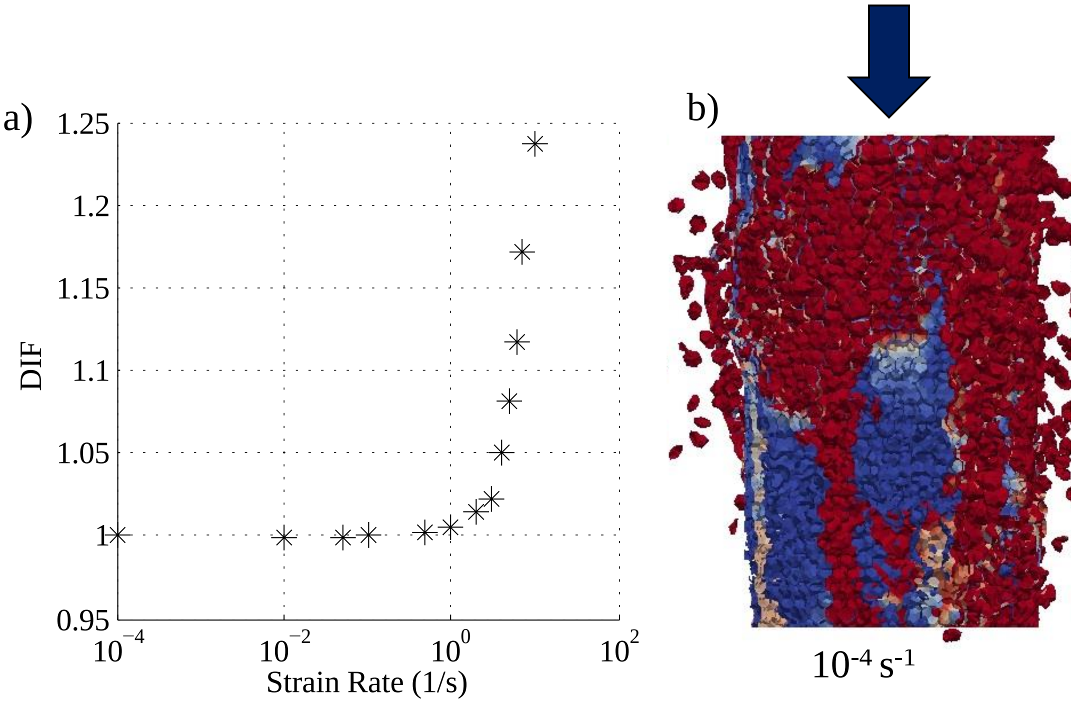
$$\mathrm{DIF}_c = \frac{f_c^{dyn}}{f_c'} = \mathrm{DIF}^* + \frac{f^{in}}{f_c'}$$

and

$$\mathrm{DIF}_t = \frac{f_t^{dyn}}{f_t'} = \mathrm{DIF}^* + \frac{f^{in}}{f_t'} \approx \mathrm{DIF}^* + 10\frac{f^{in}}{f_c'}$$

Inertia and Crack Patterns Effects

- Apparent rate-effect phenomena captured automatically

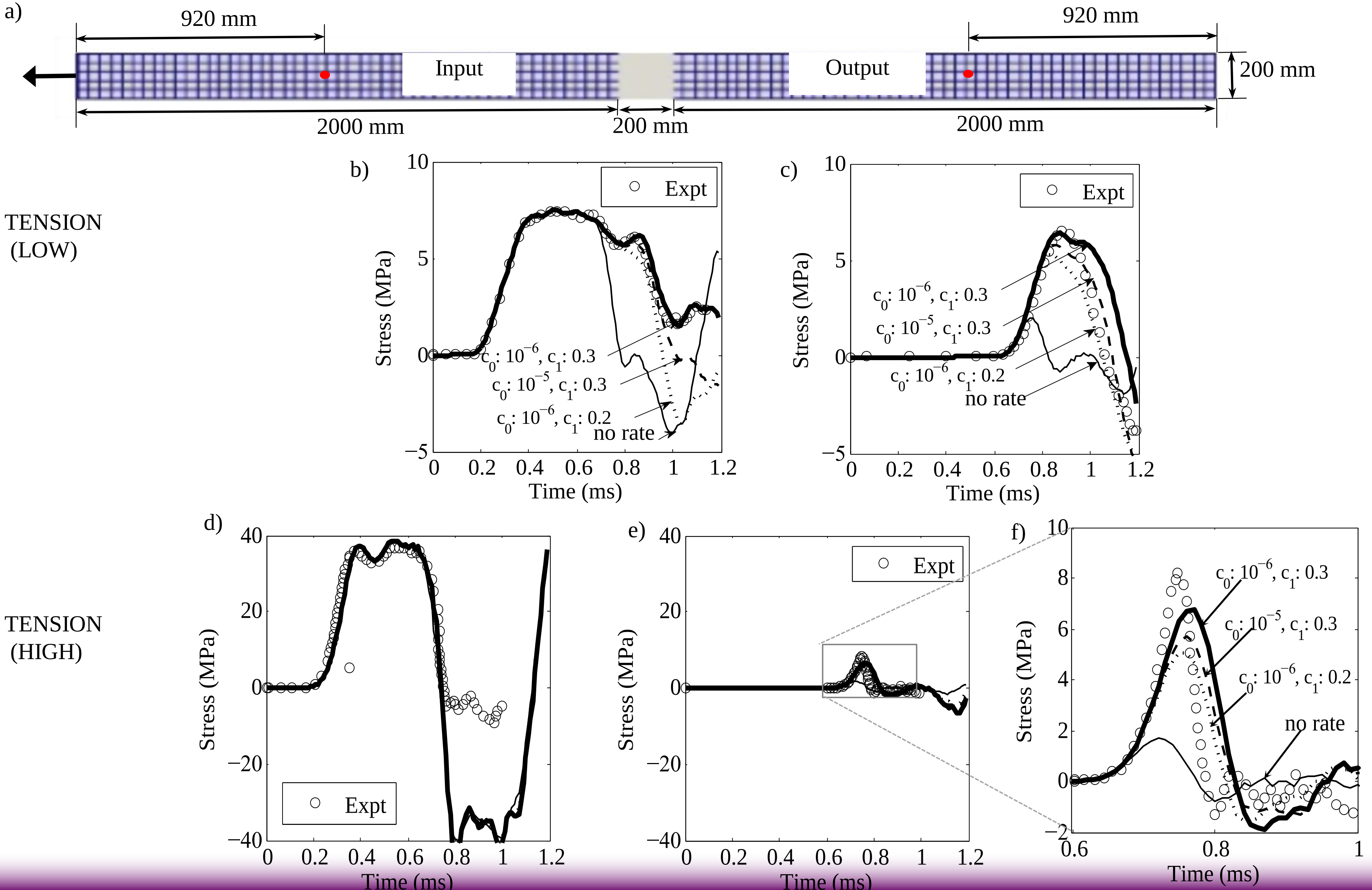


Smith J et. al.. “Discrete Modeling of Ultra High Performance Concrete with Application to Projectile Penetration” 65 (2014) 13 – 32, *Int J of Imp Eng.*

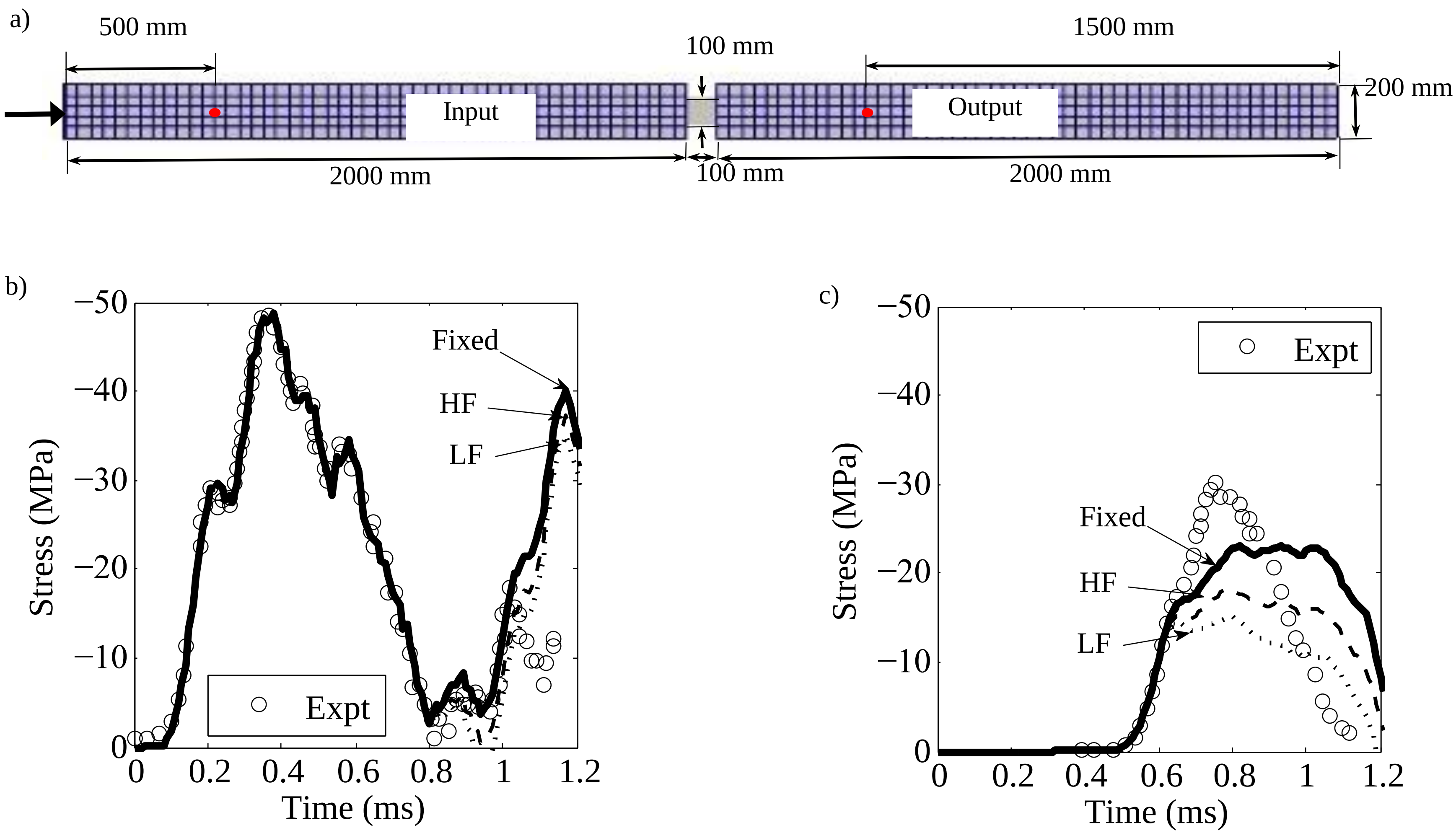
Hopkinson Bar Test - Tension

TENSION
(LOW)

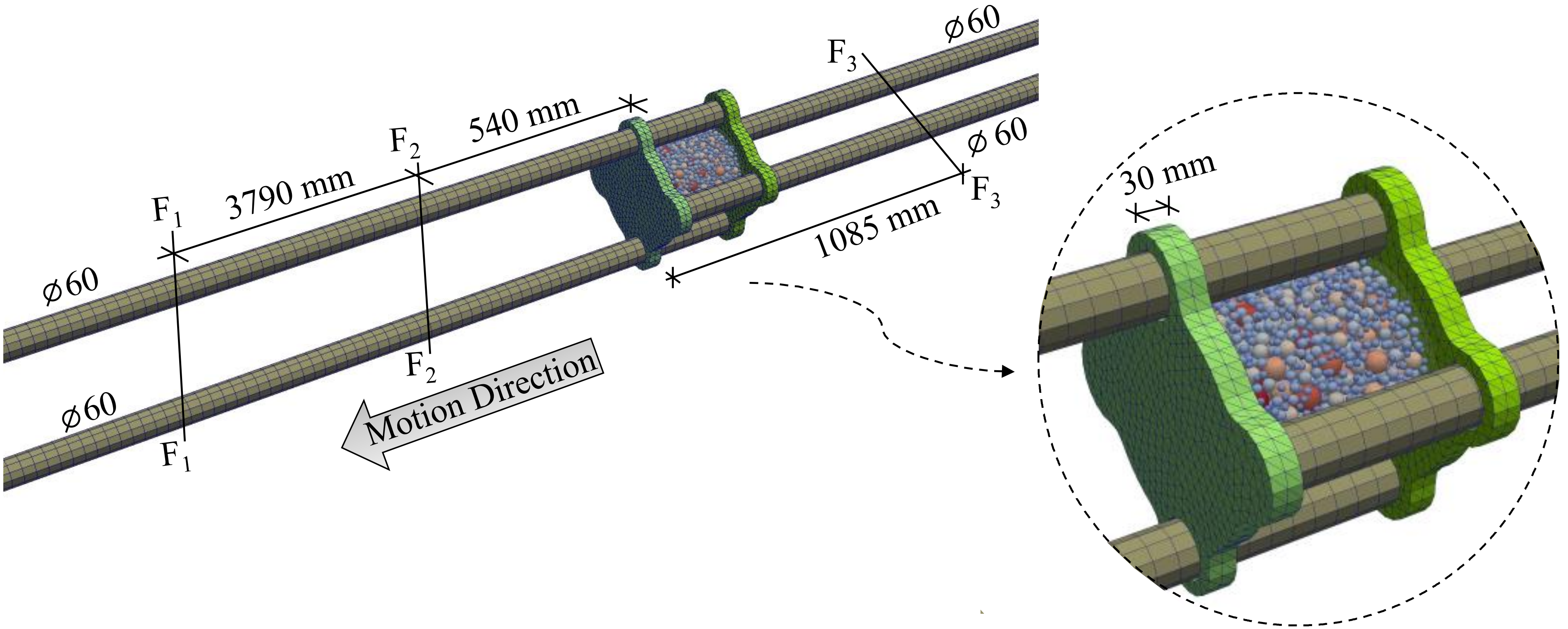
TENSION
(HIGH)



Hopkinson Bar Test - Compression



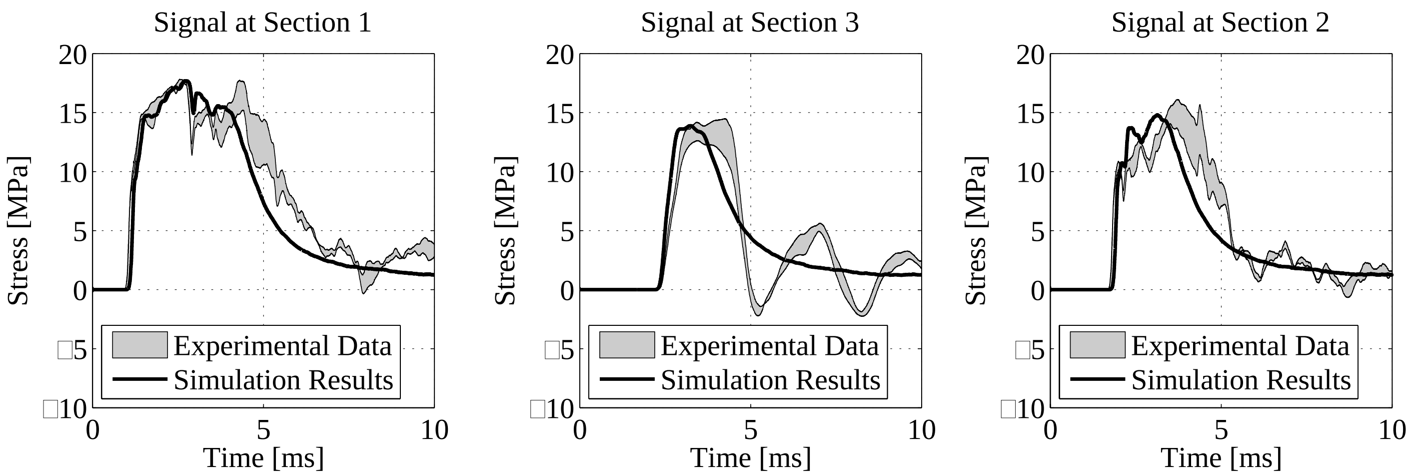
Compression with Twins Bars



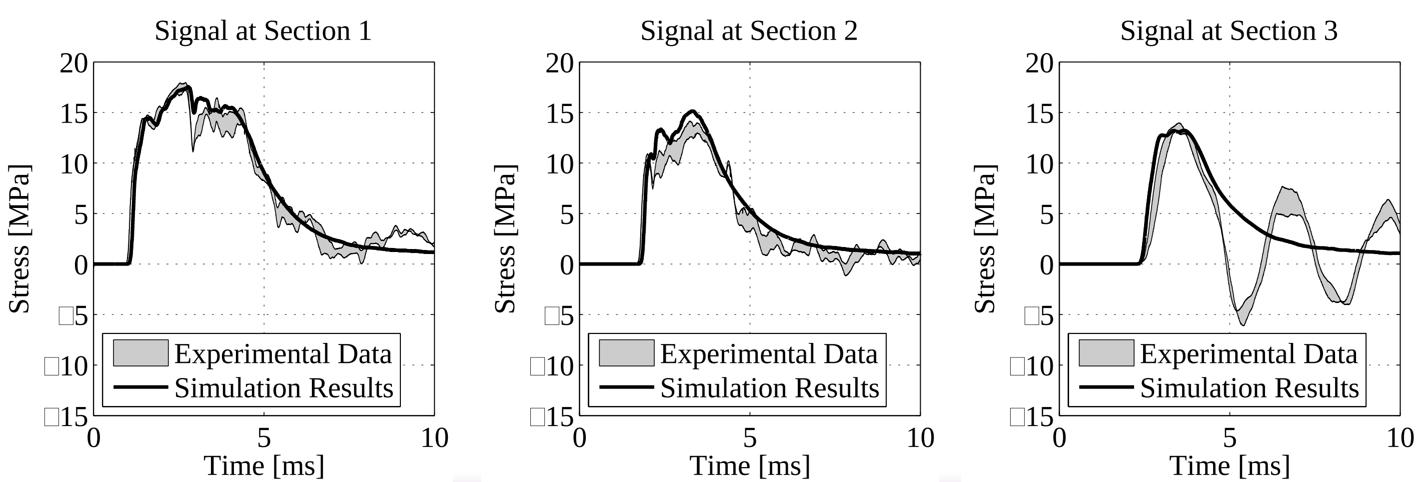
Tests on standars and dam concrete mixes

Compression with Twins Bars, Cont

Small Cylindrical Specimen (Dam concrete):

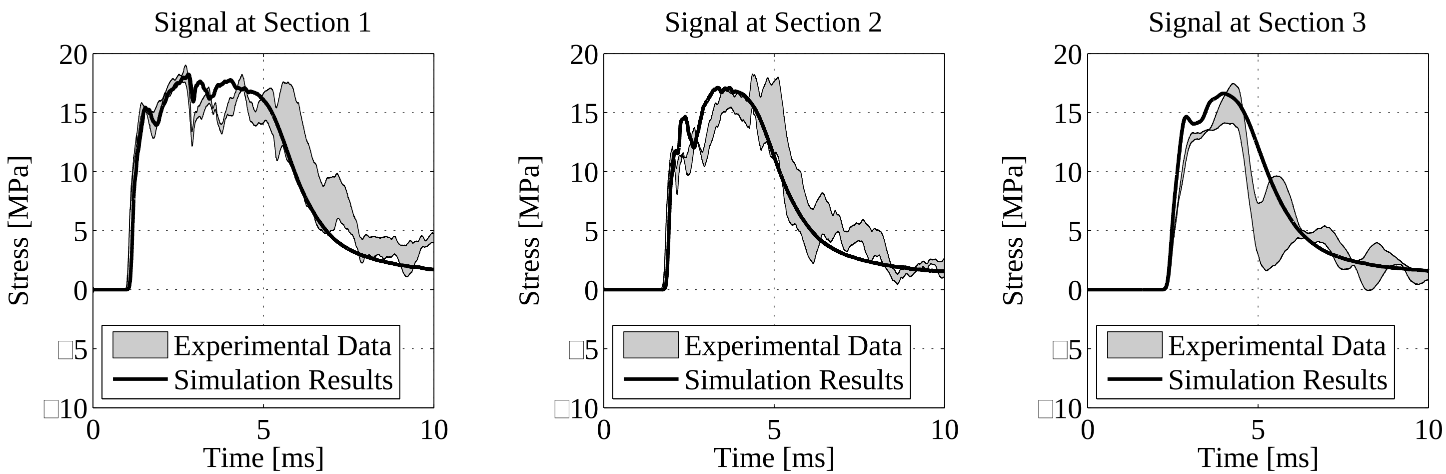


Large Cylindrical Specimen (Dam concrete):

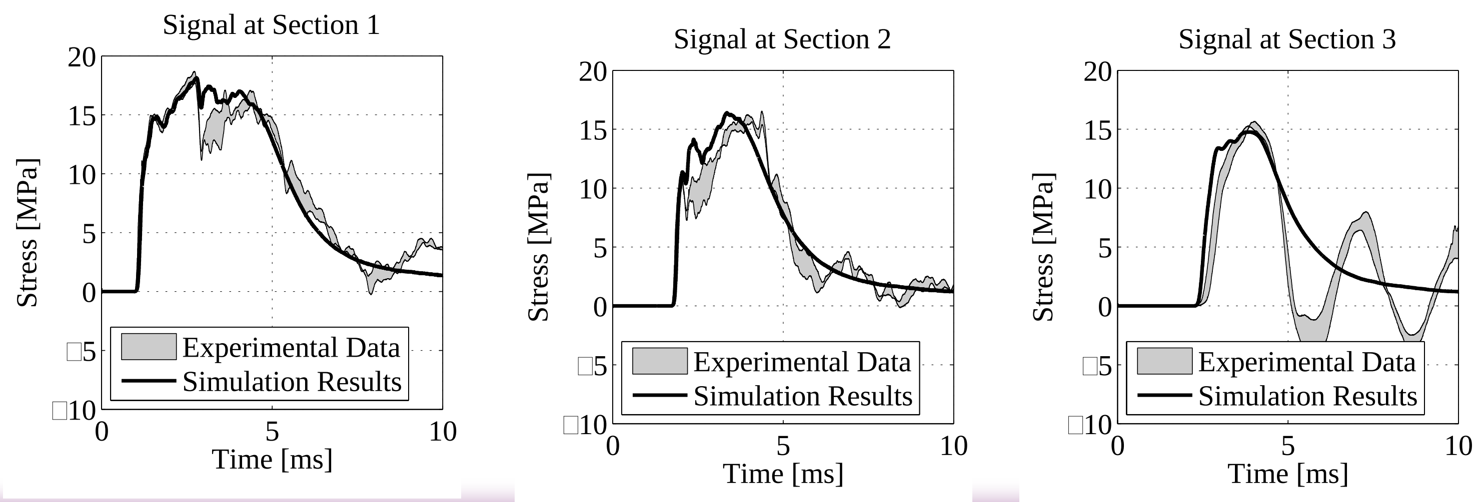


Compression with Twins Bars, Cont

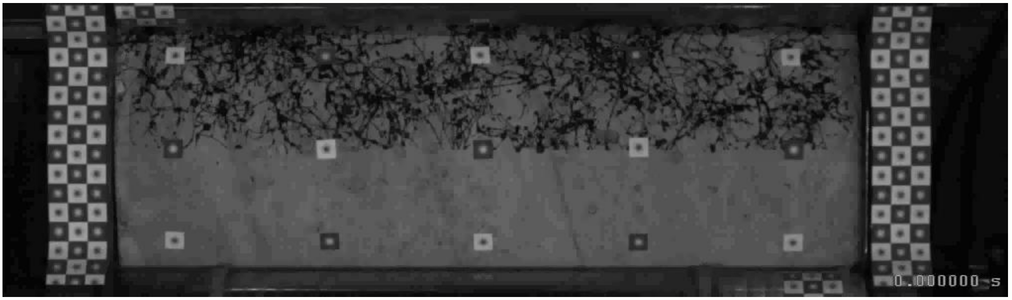
Small Cylindrical Specimen (Standard concrete):

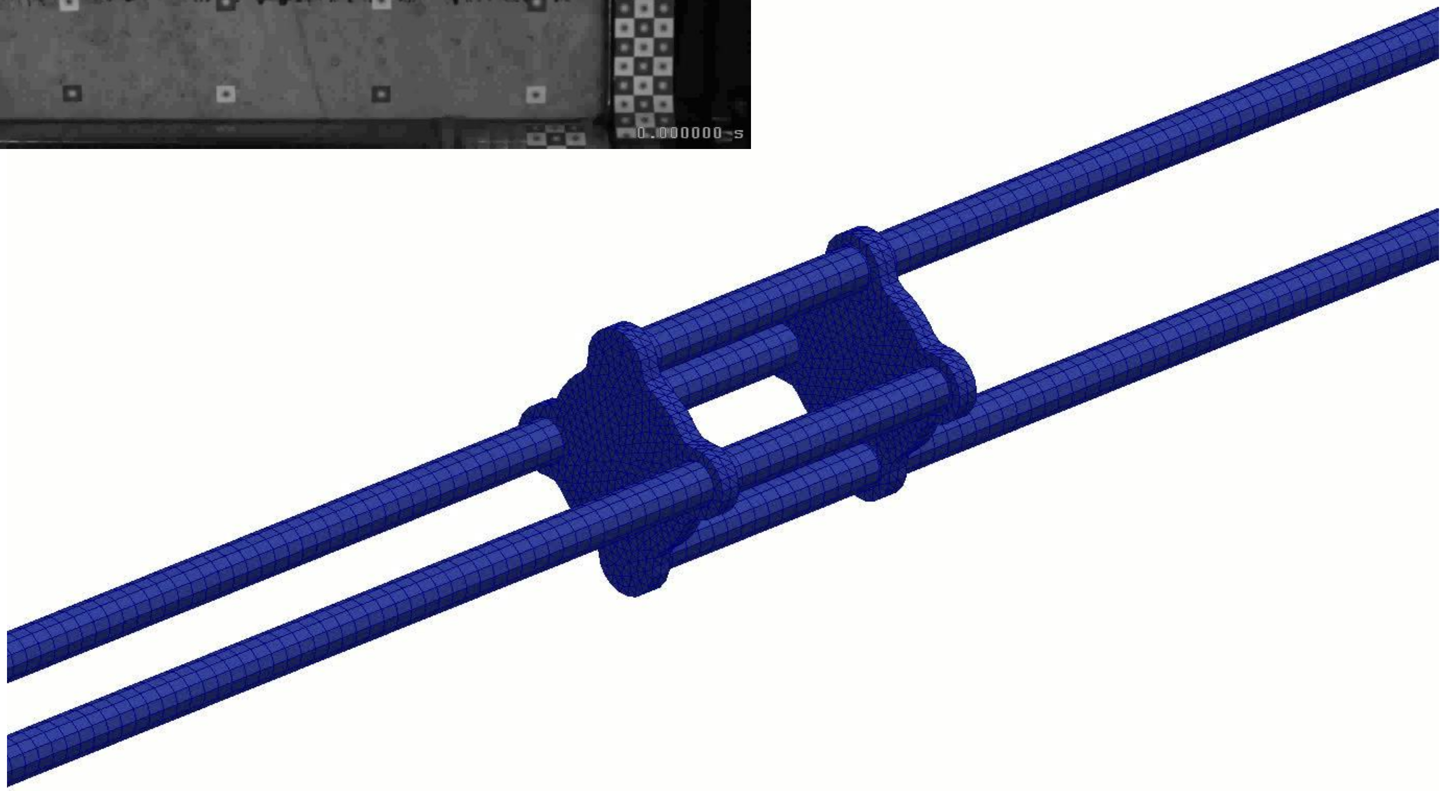


Large Cylindrical Specimen (Standard concrete):

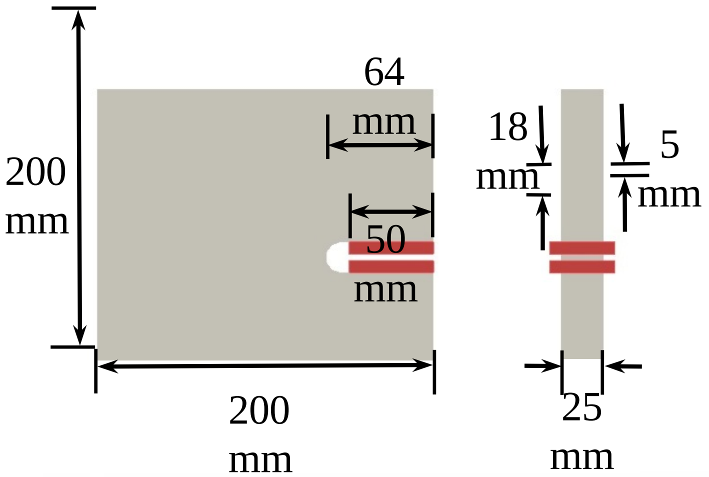


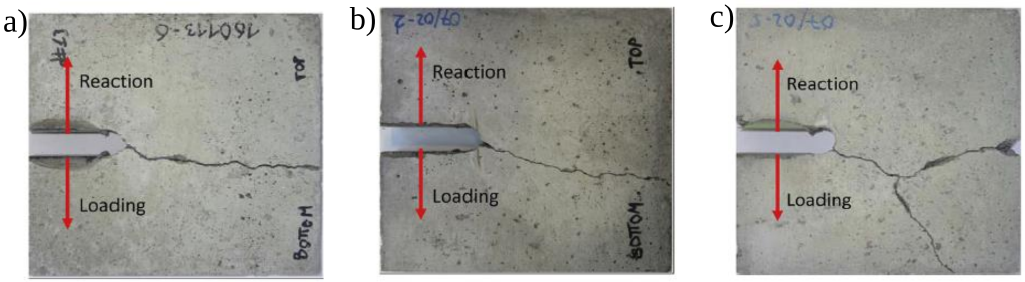
Compression with Twins Bars, Cont



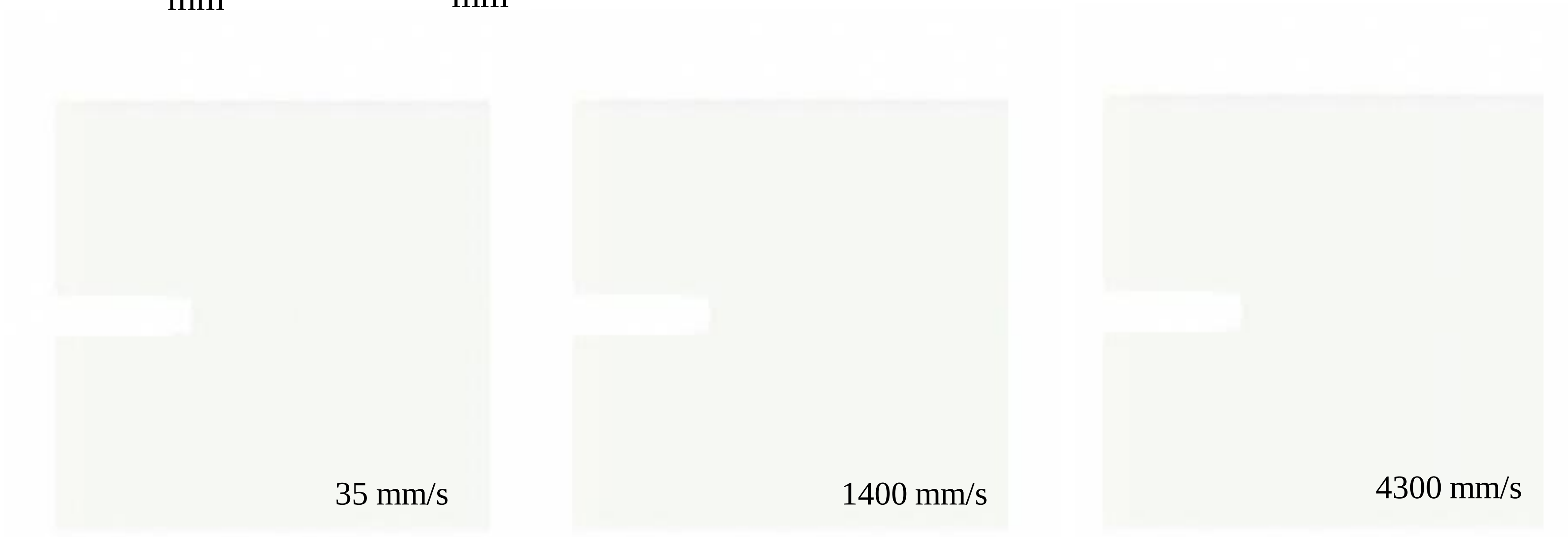


Dynamic Concrete Tension Test

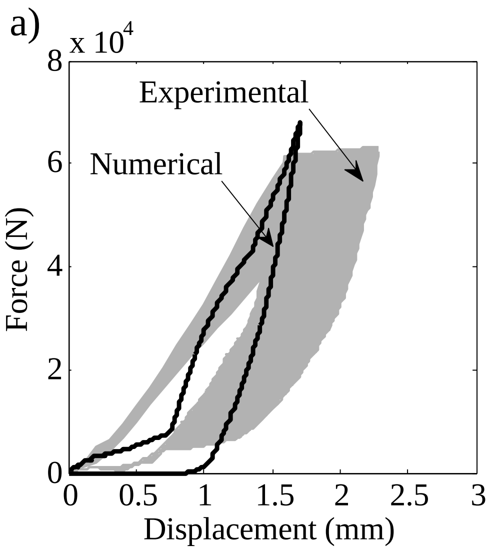


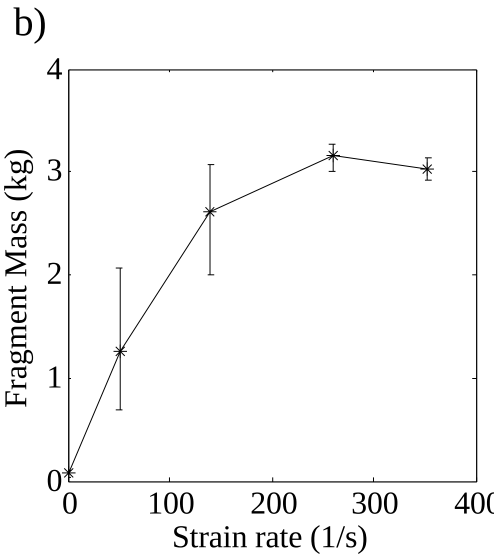


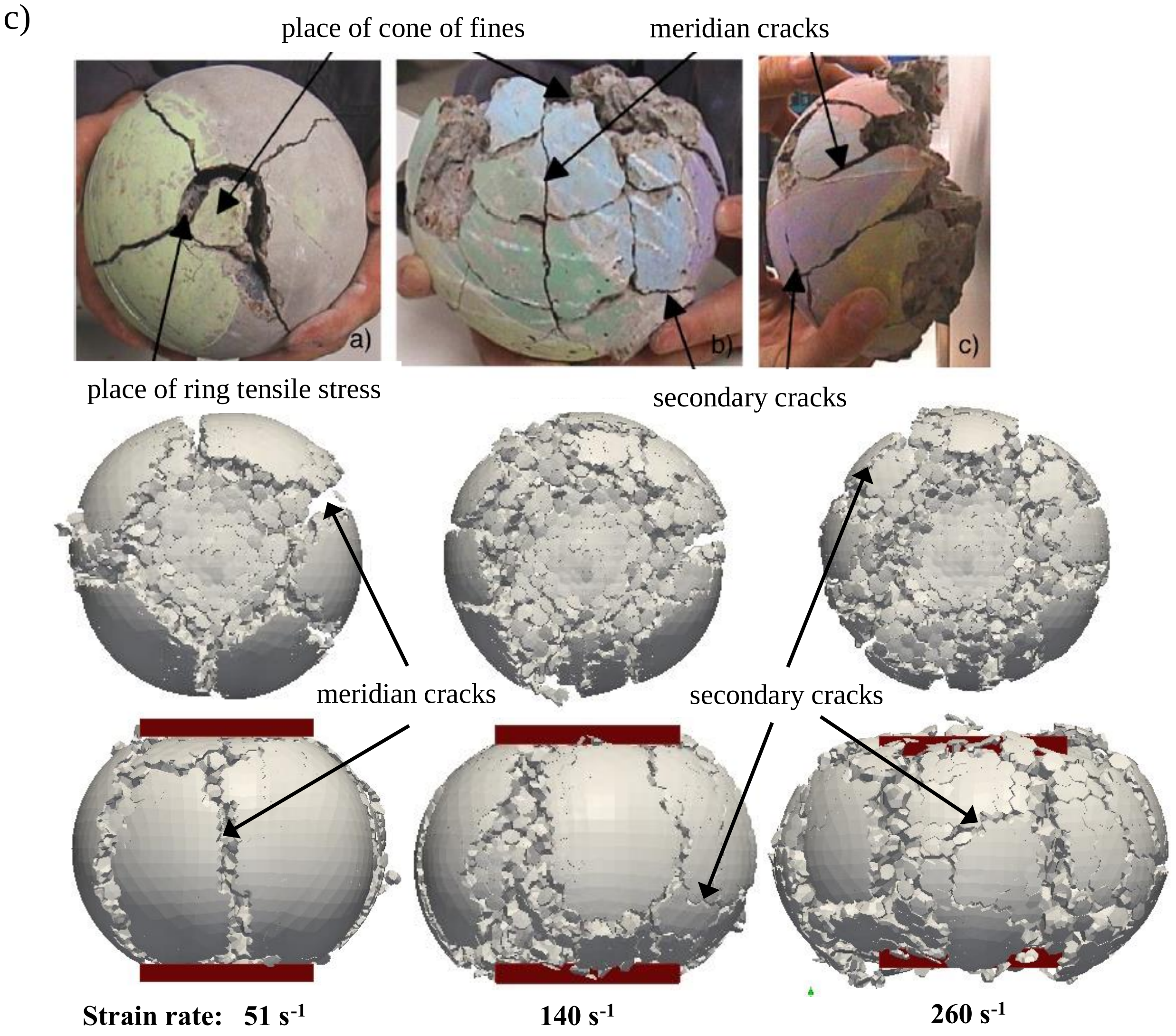
35 mm/s 1400 mm/s 4300 mm/s



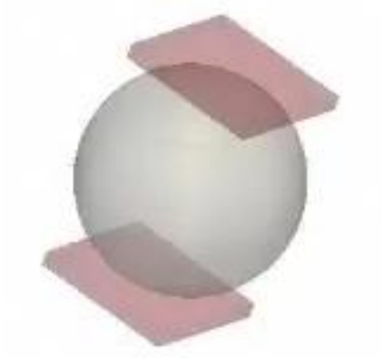
Concrete Ball Impact Test



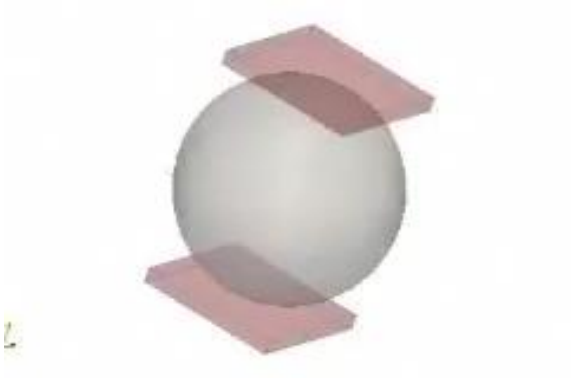




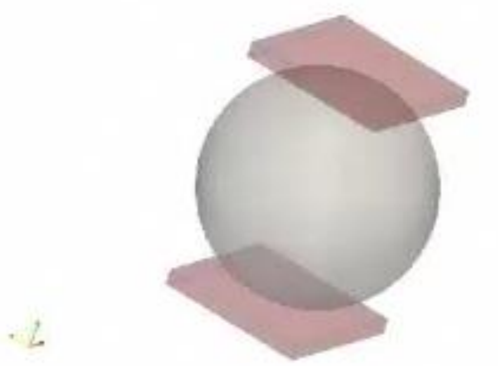
Concrete Ball Impact Test



1.1E-5 s^{-1}

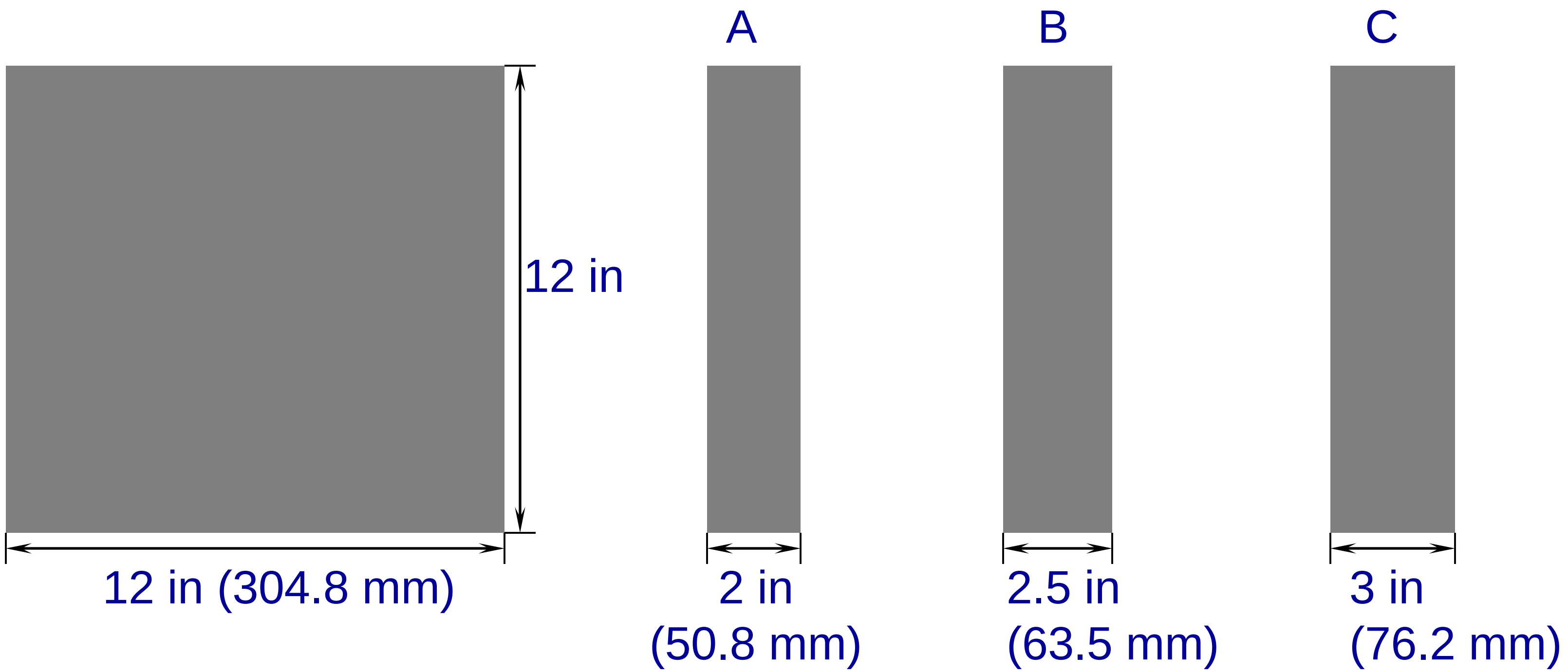


140 s^{-1}

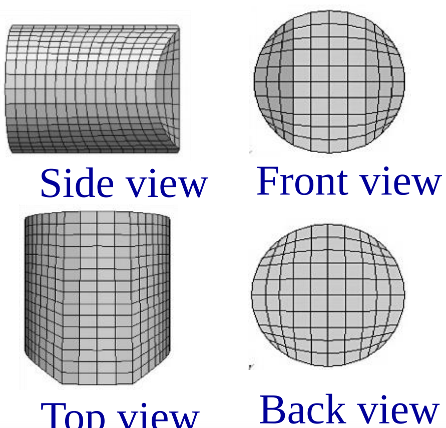


353 s^{-1}

Penetration of UHPC Panels



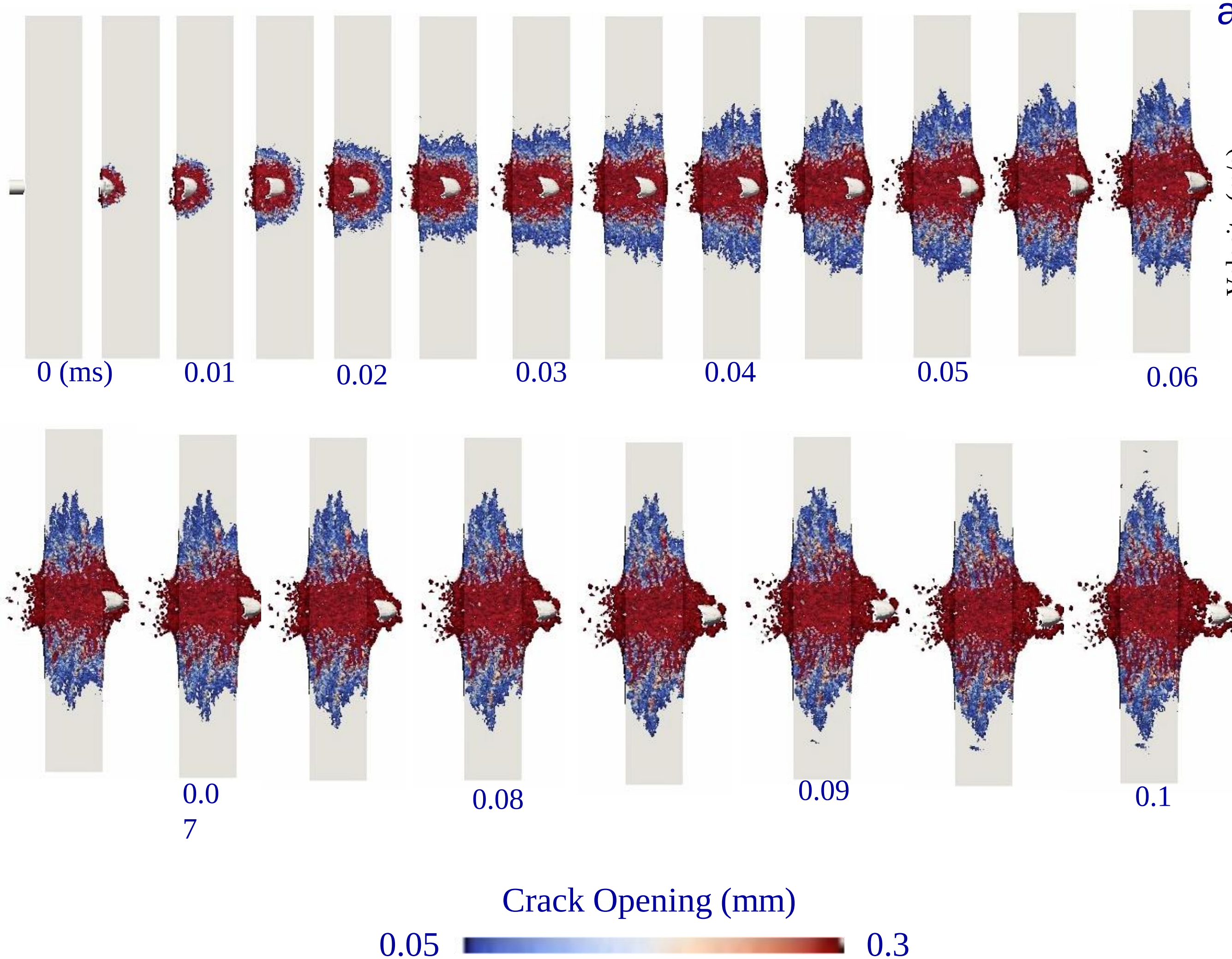
- **(FSP) projectile:**
  - **4340-H steel**
  - **Yield strength = 930 MPa**
  - **Diameter = 12.5 mm**
  - **Length = 14.8 mm**



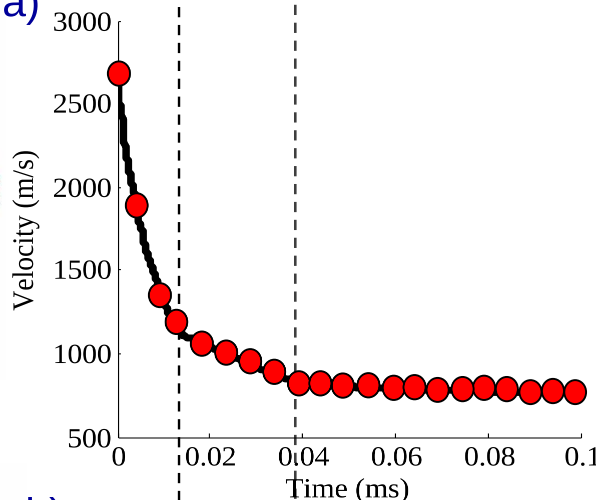
Smith J et. al.. "Discrete Modeling of Ultra High Performance Concrete with Application to Projectile Penetration" 65 (2014) 13 – 32, *Int J of Imp Eng.*

Damage Evolution

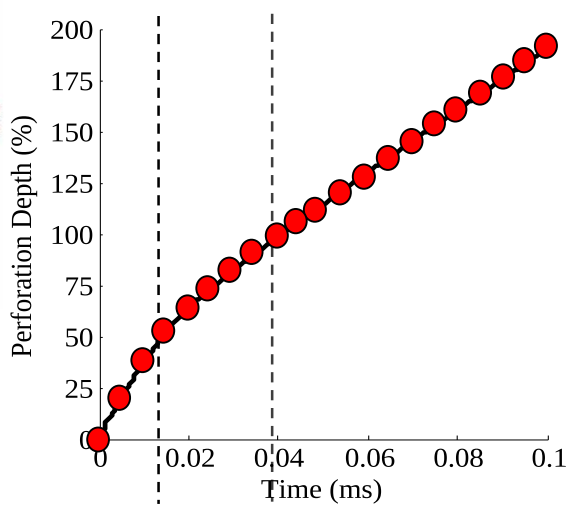
Progression of projectile penetration for CORTUF-Plain size A



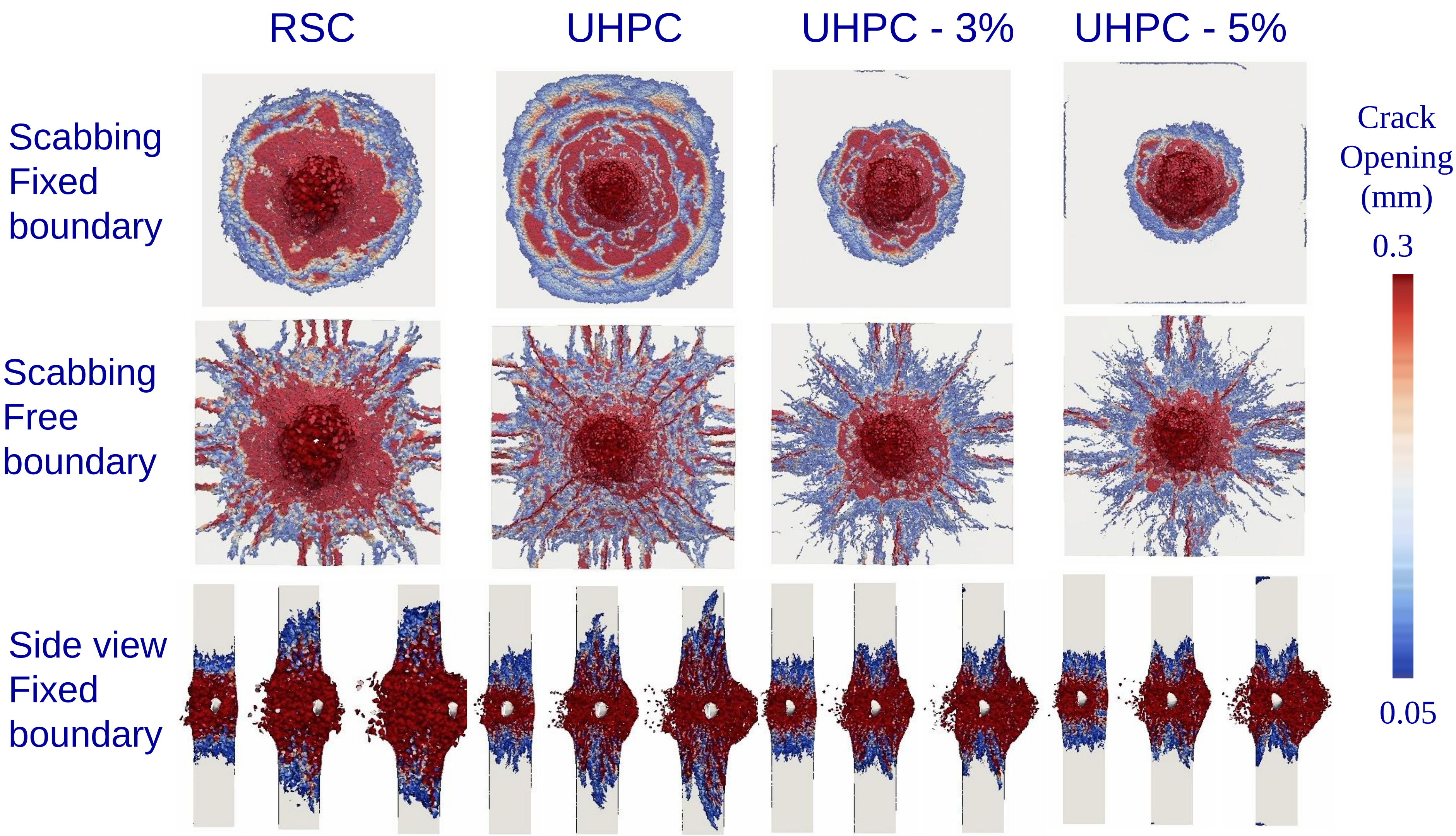
a)



b)



Effect of Fiber Content



Penetration of Regular Strength Concrete

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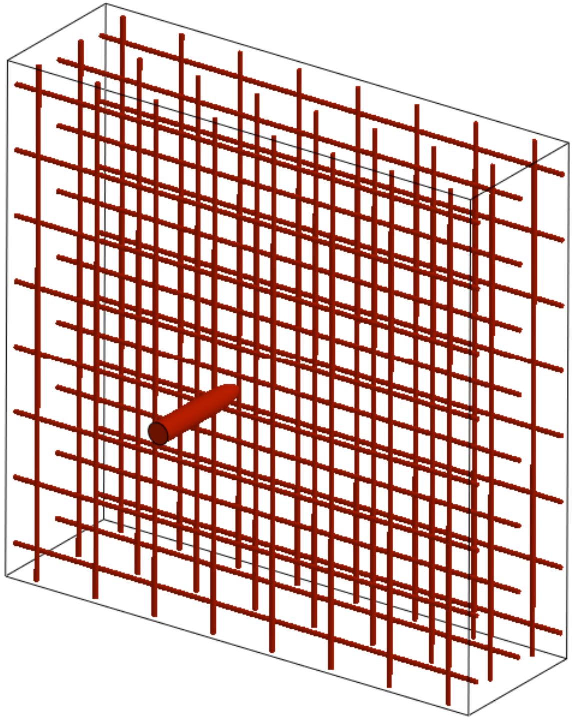
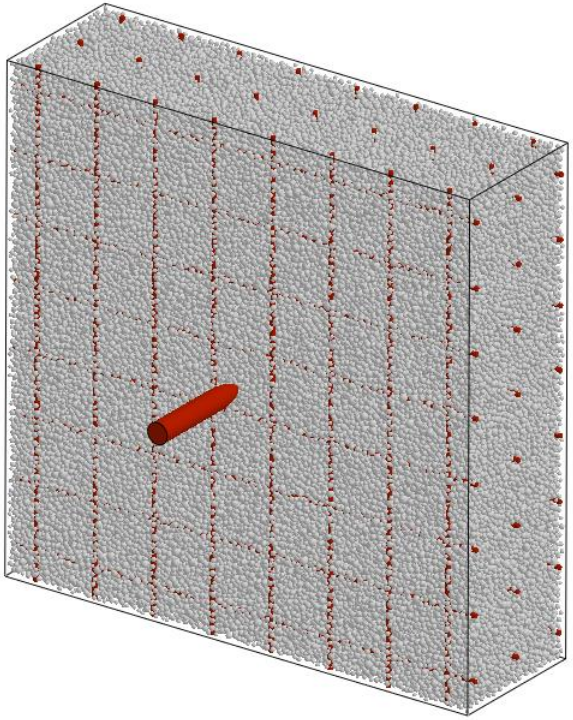
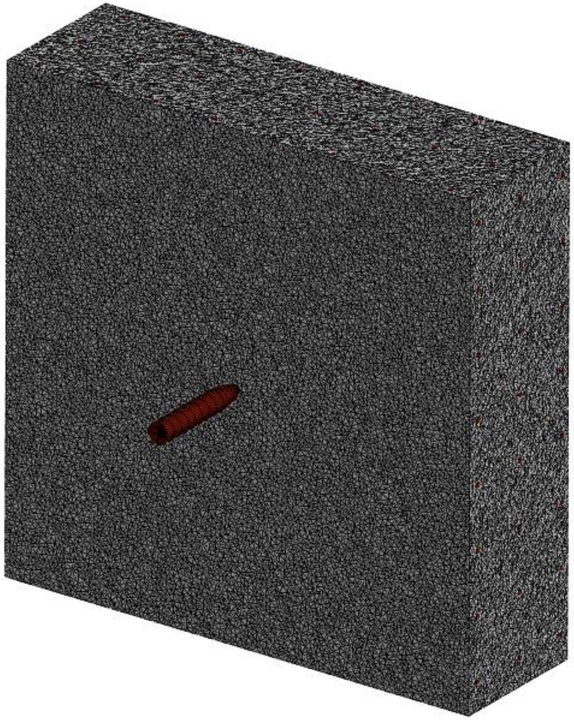
- Experimental data (Hanchak et al. 1992) relevant to impact of steel projectiles against lightly reinforced concrete slabs
- Projectile of mass m=0.5 kg and diameter d = 25.4 mm
- Slab 610 x 610 x 178 mm
- Concrete Young Modulus 20000 MPa
- Concrete Strength f'_c = 48 MPa
- Impact velocity from 300 m/s to 1000 m/s

Full Meso-Scale Simulations





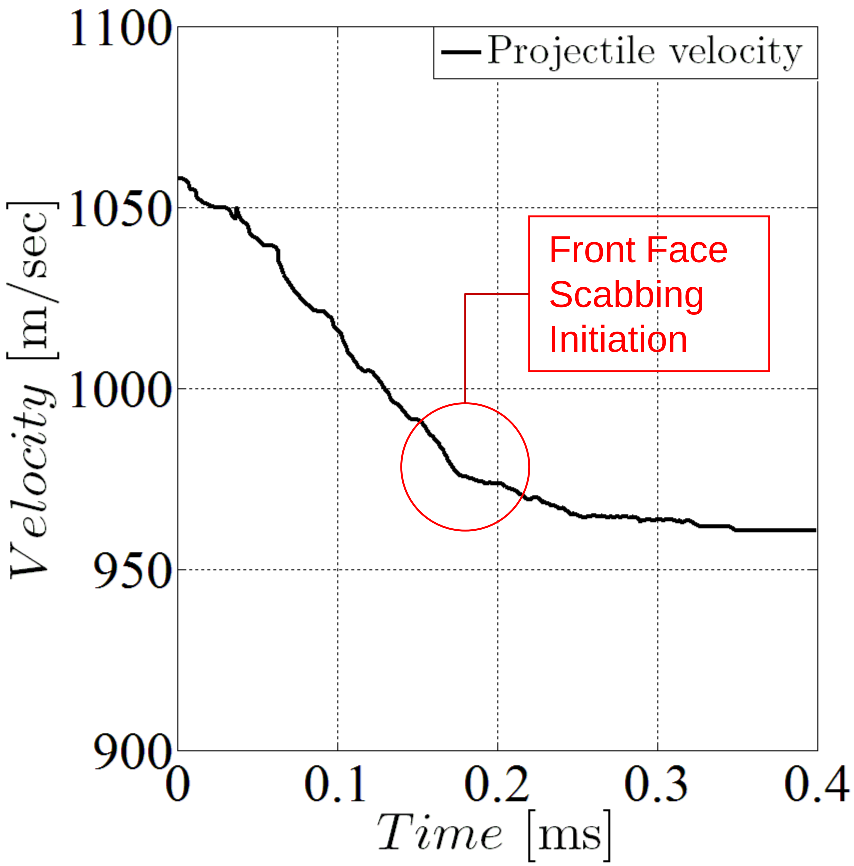


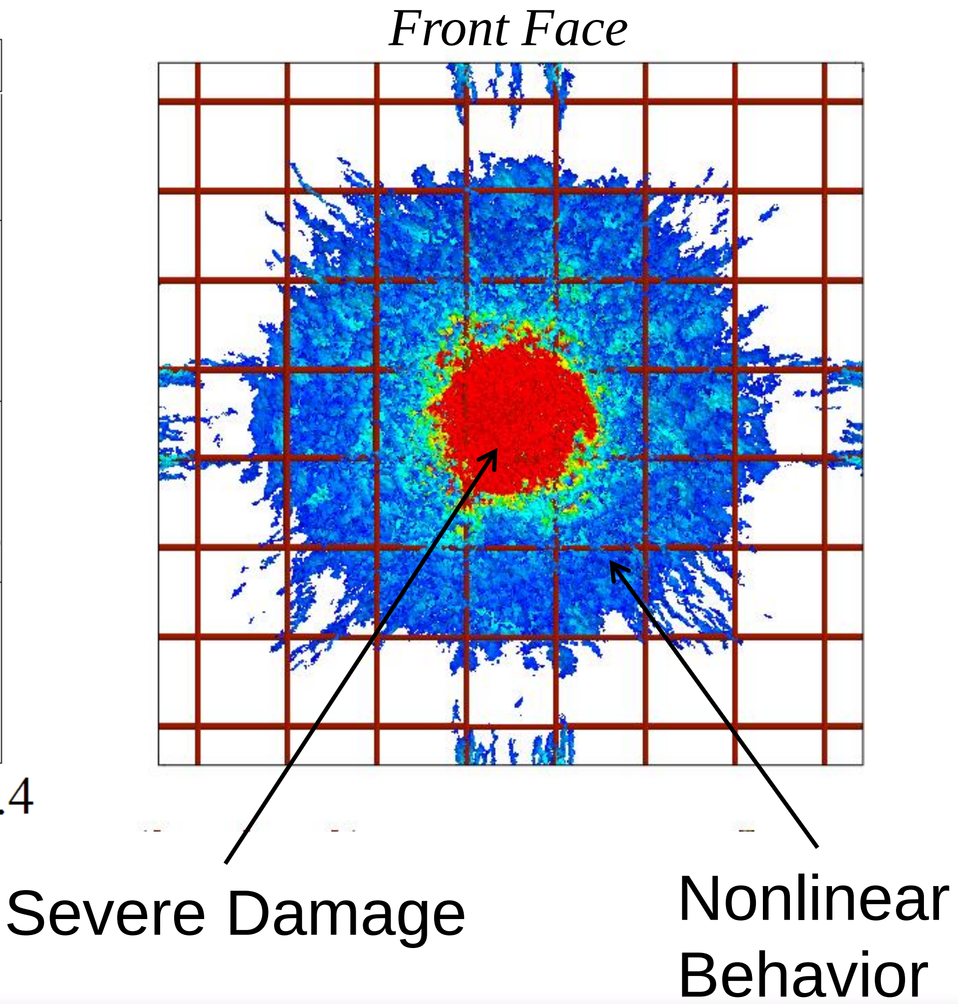
Steel reinforcement
diameter = 0.569 cm
spacing = 7.62 cm

208,967 nodes
1,253,802 dofs

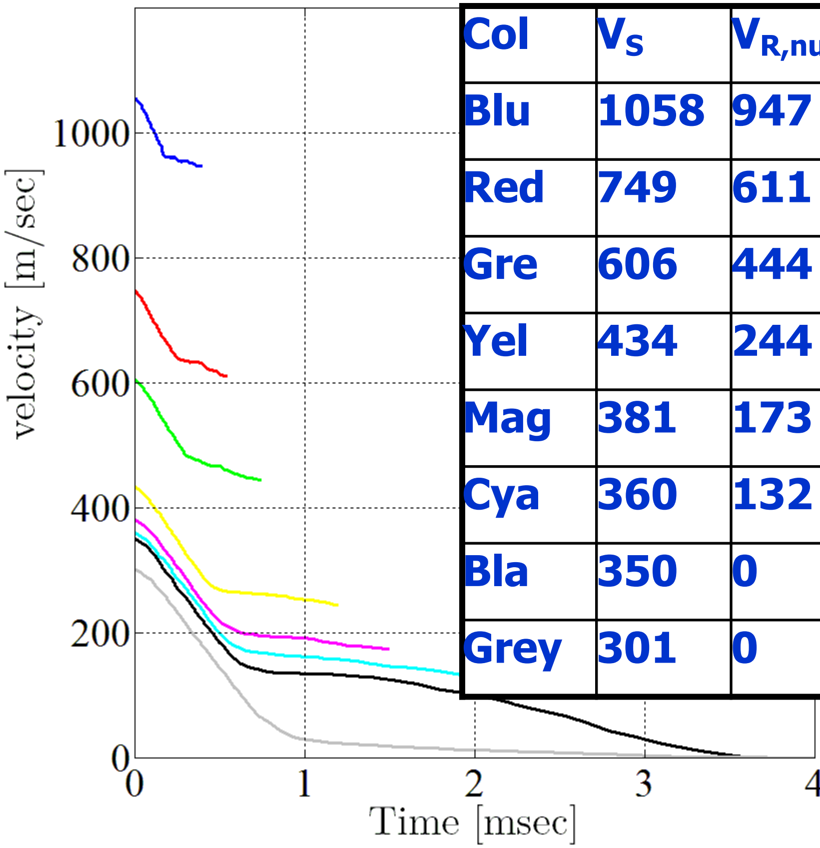
1,229,348 LDPM tets

Full Meso-Scale Simulations: Results

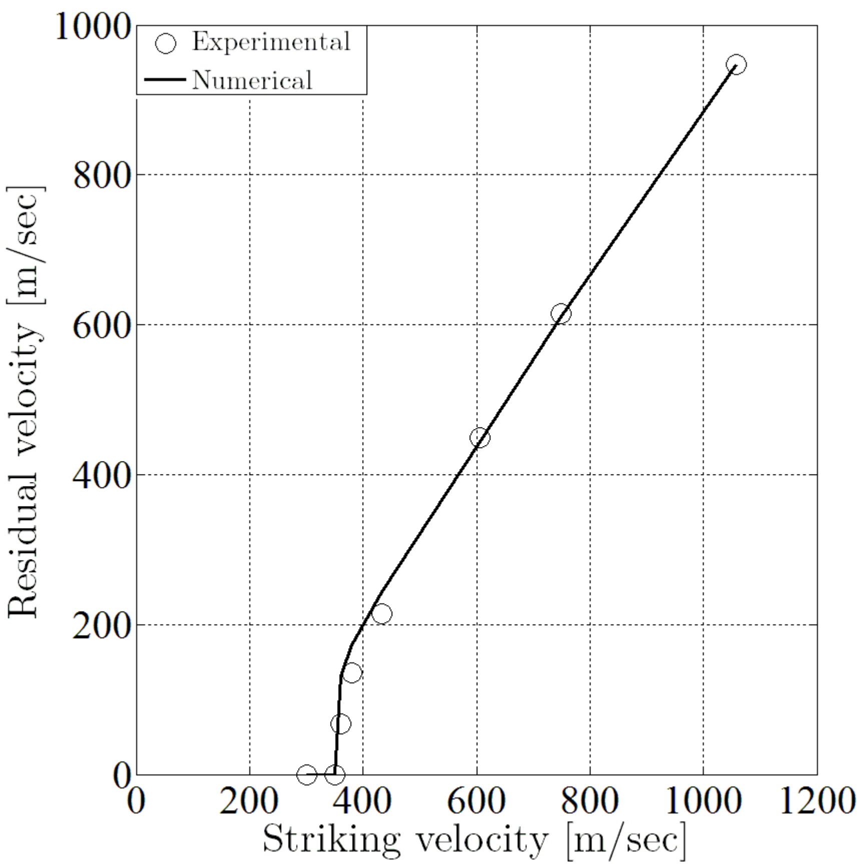




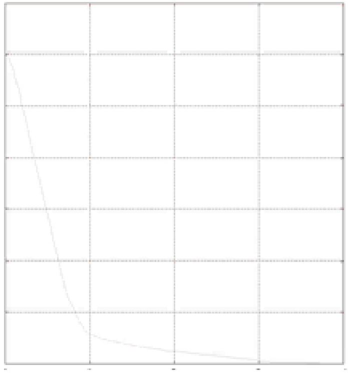
Comparison with Experiments

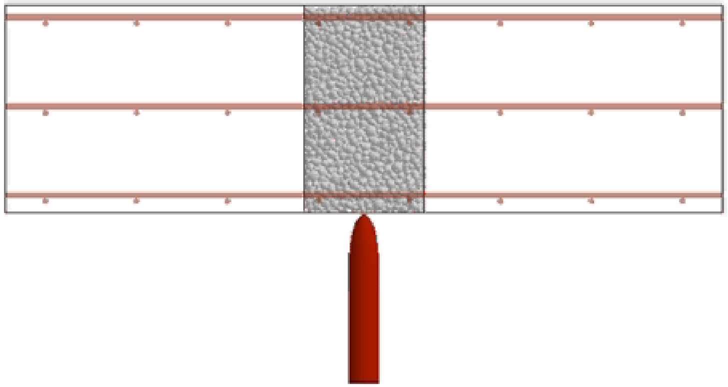


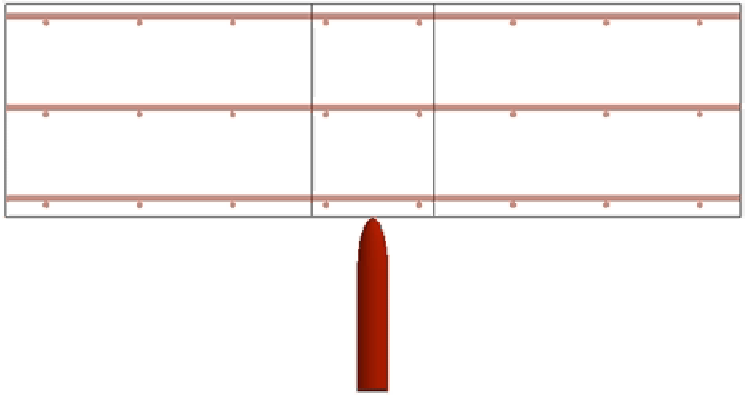
Col	V_S	$V_{R,num}$	$V_{R,exp}$
Blu	1058	947	947
Red	749	611	615
Gre	606	444	449
Yel	434	244	214
Mag	381	173	136
Cya	360	132	67
Bla	350	0	0
Grey	301	0	0

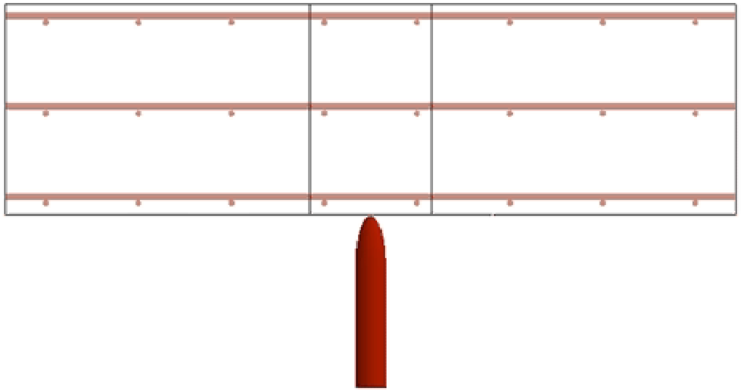


Animation: Ballistic Limit (~350 m/s)

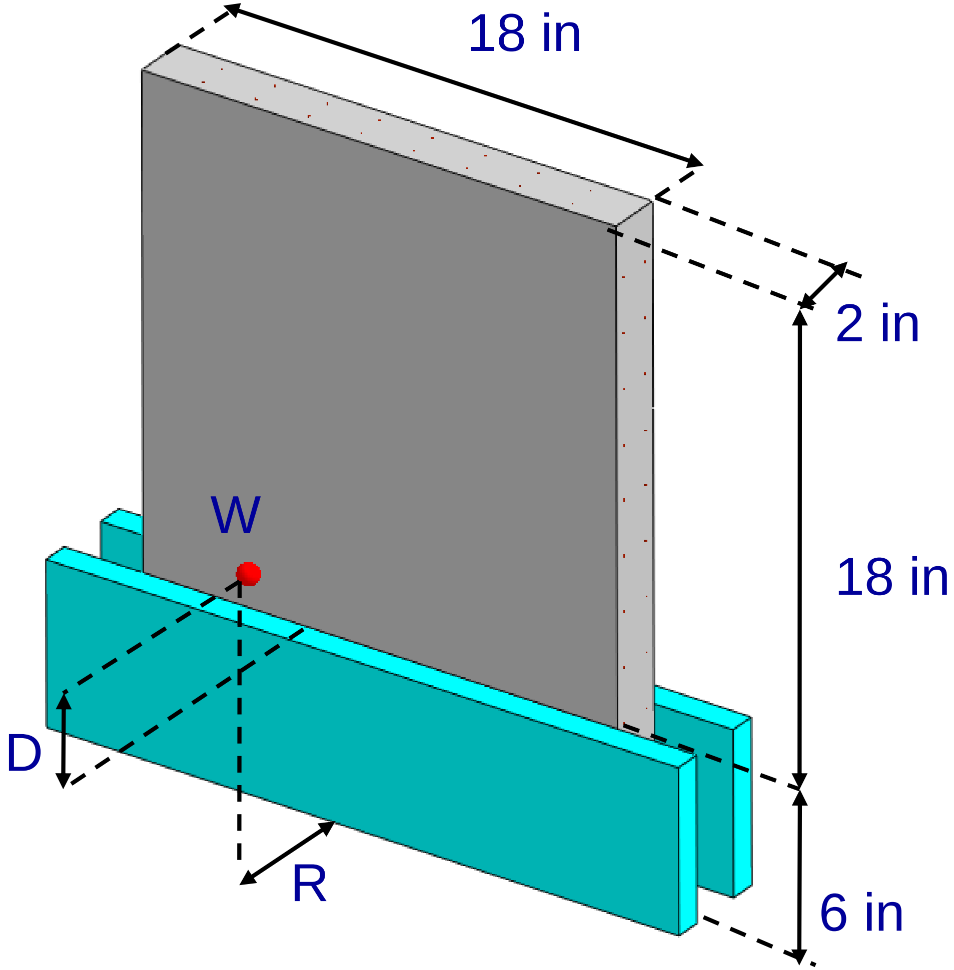


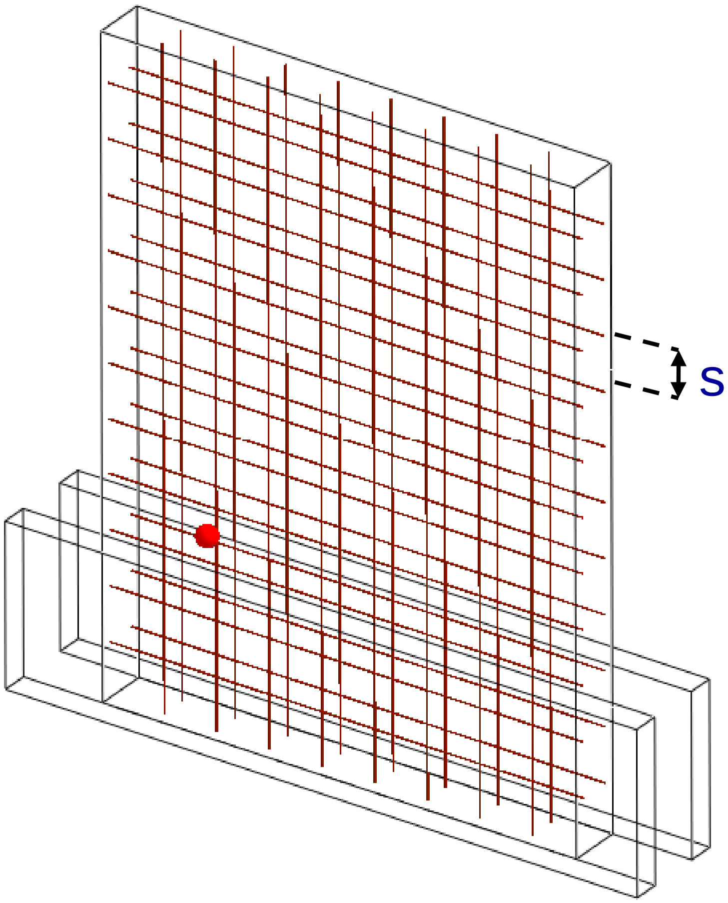






Blast Simulations: Geometry 1





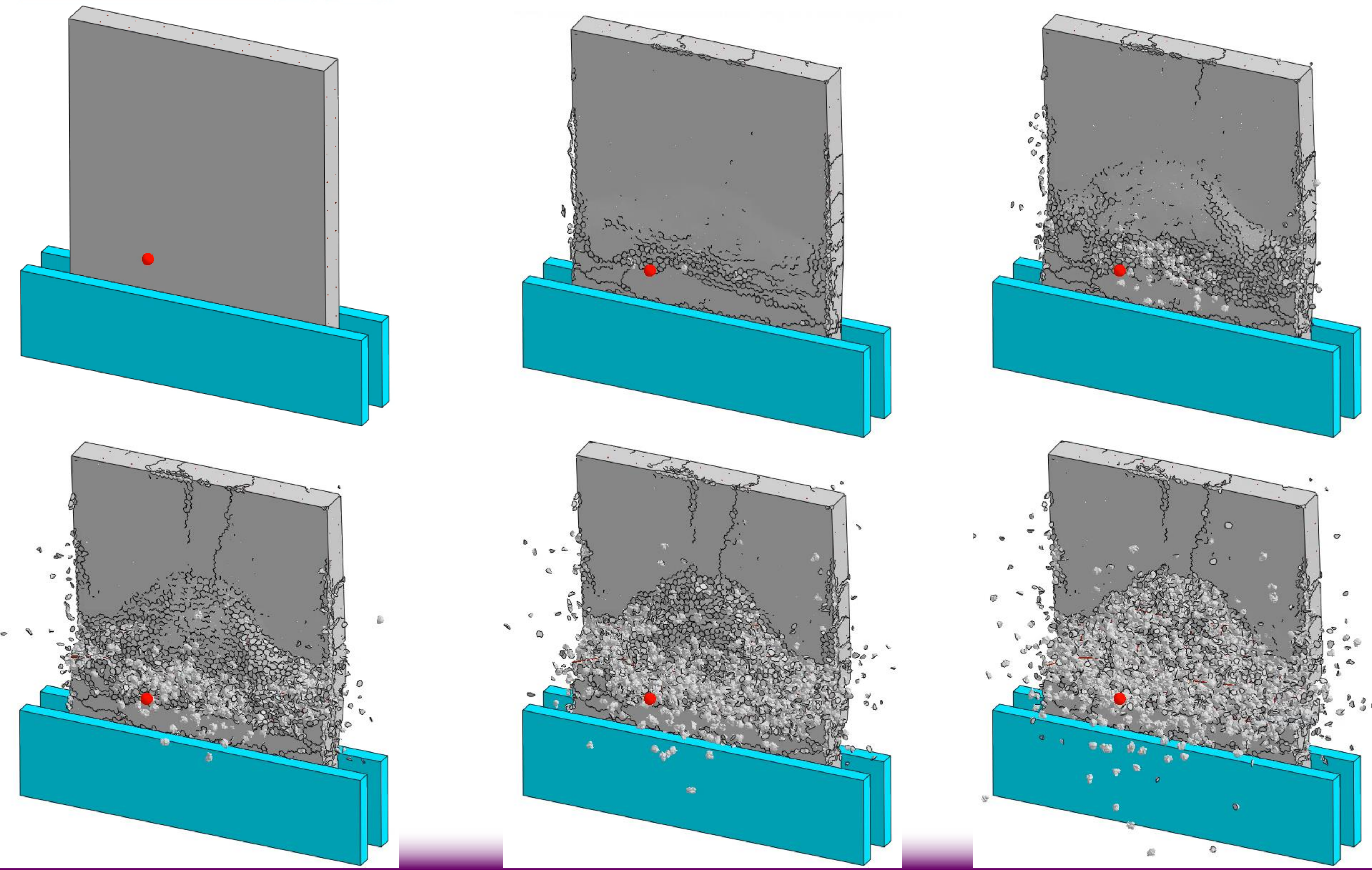
Simulated Tests

NORTHWESTERN UNIVERSITY

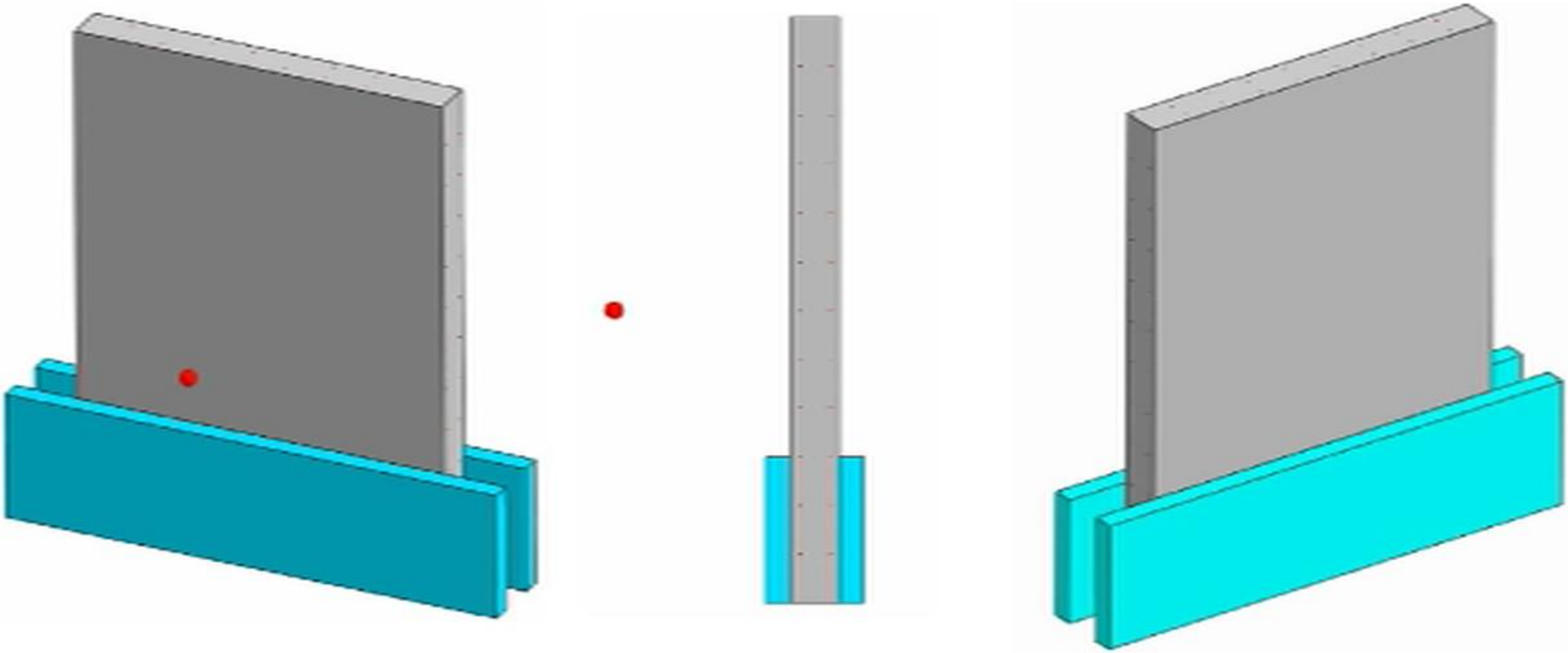
Test No.	Rebar spacing s (mm)	W C4 (kg)	R (m)	D (mm)
1	50.8	0.454	0.183	152
2	25.4	0.454	0.183	152
3	50.8	0.454	0.183	152
4	25.4	0.454	0.183	152
5	50.8	0.227	0.152	229

- Compressive Strength=26.7 MPa
- Experimental Data from “Explosive fragmentation of dividing walls”, Report ARLCD-CR-81018;
- Blast-reflected pressures computed using US Army, US Navy, US Air Force, 1990. “Structures to resist the effects of accidental explosions”. Technical report TM5-1300, NAVFAC P-397, AFR 88-22 and Hyde, D.W., 1992. “CONWEP, Conventional Weapons Effects Program.” Technical report, US Army Engineer Waterways Experiment Station, Vicksburg, MS.

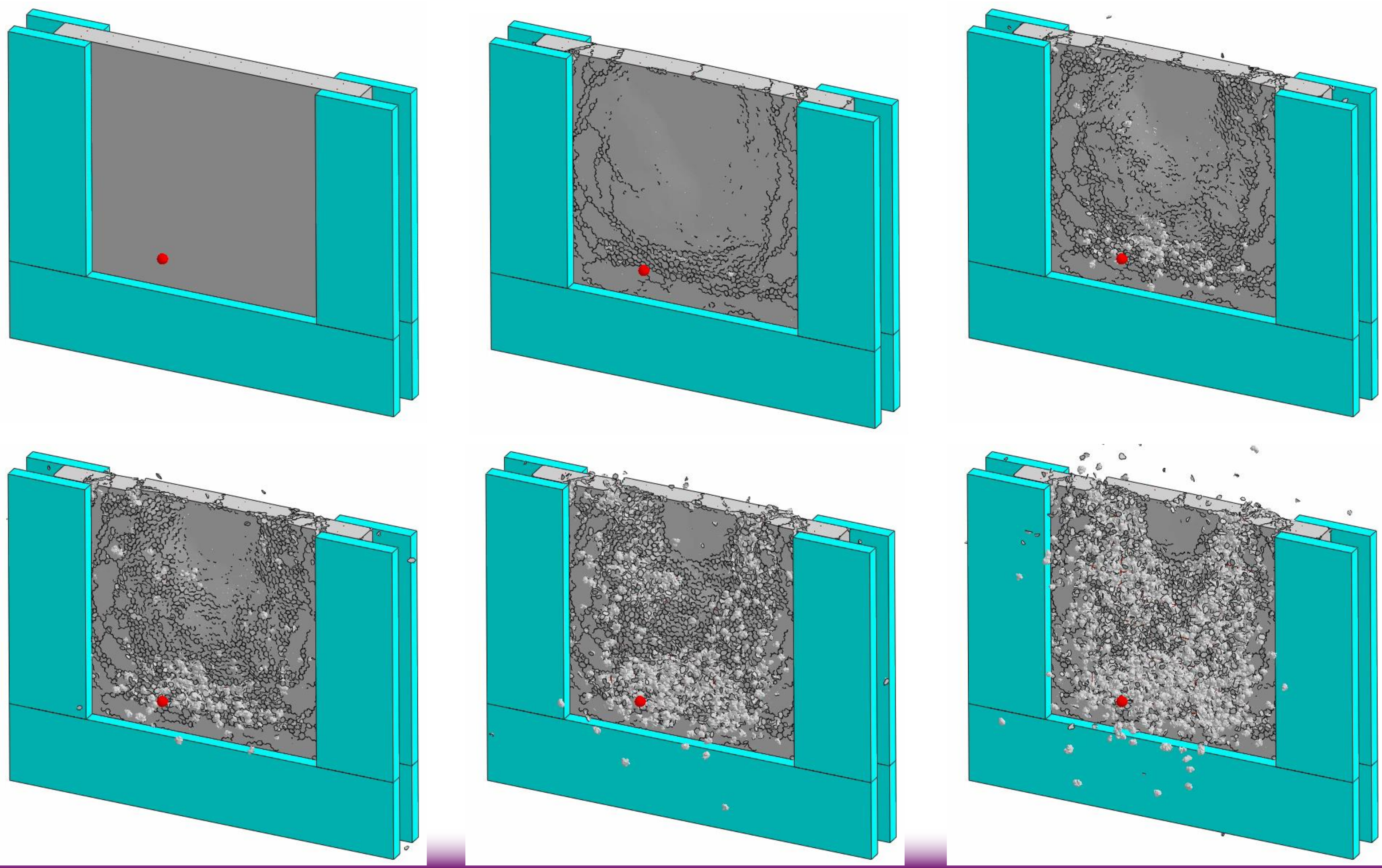
Results: Test 1



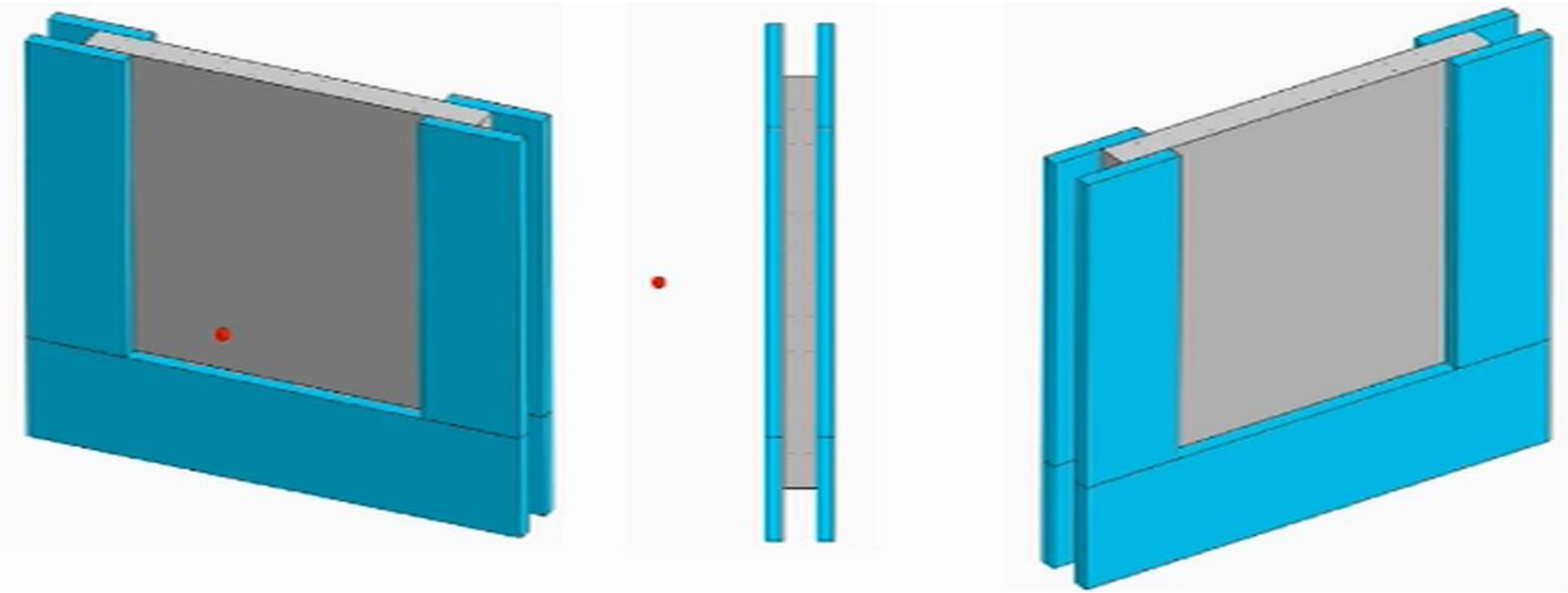
Animation Test 1



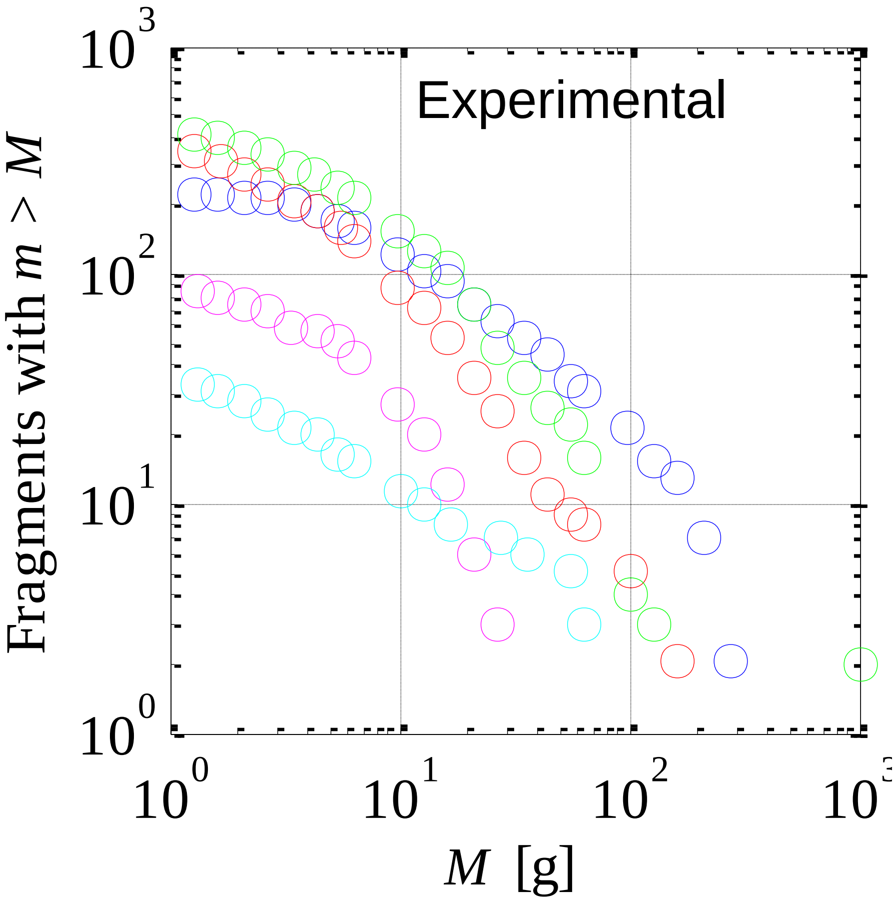
Results: Test 3

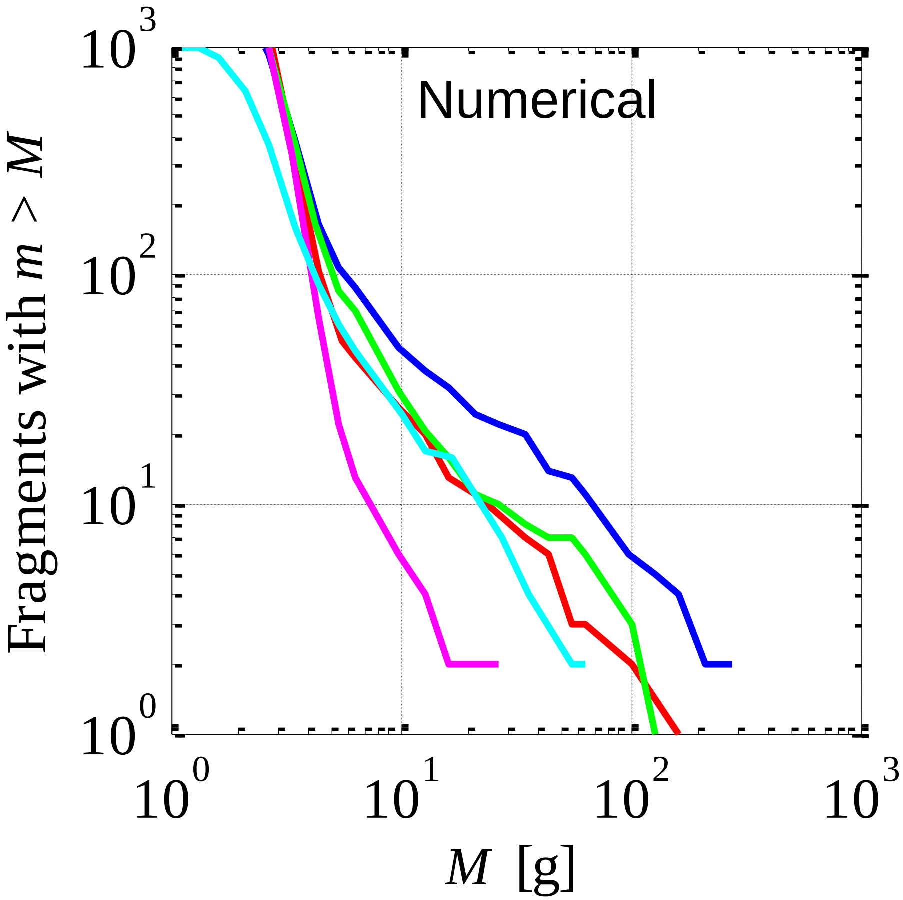


Animation Test 3



Fragment Distributions





Blue = test 1; Red = test 2; Green = test 3;

Pink = test 4; Cyan = test 5.

MARS – Multiscale-multiphysics Analysis of the Response of Structures

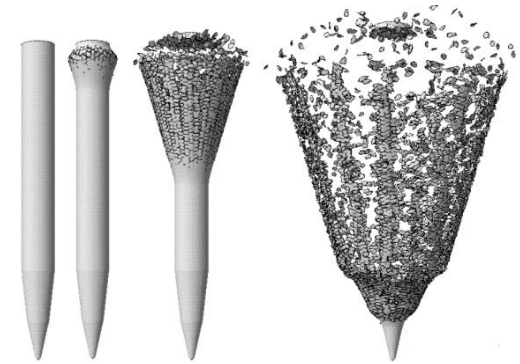
http://mars.es3inc.com

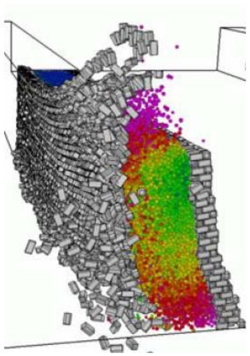
Grenoble, France | Oct 21, 2016

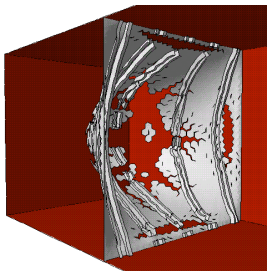
The MARS Solver

MARS (Modeling and Analysis of the Response of Structures) is a multipurpose object-oriented computational software for simulating the mechanical response of structural systems subjected to short duration events.

It is based on dynamic explicit algorithms and it implements all the capabilities and versatility of a general finite element code.



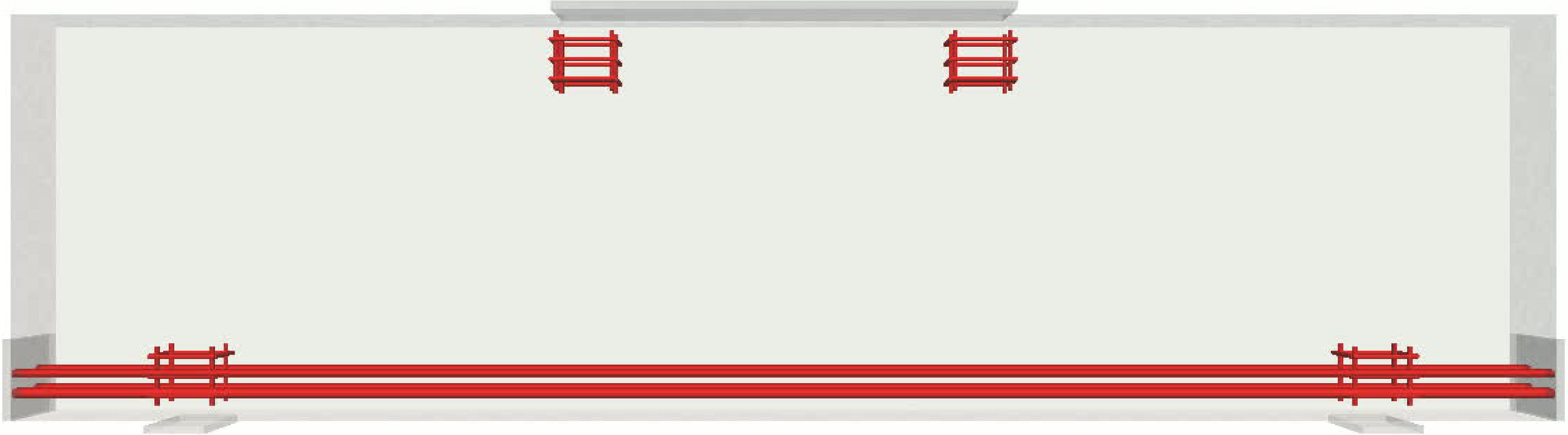




MARS Is a General Purpose Structural Dynamic Code

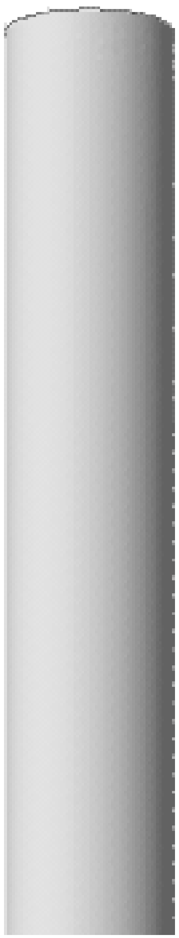
- Lattice Discrete Particle Model (LDPM) for simulations of cementitious materials
- QPH quadrilateral shell elements with physical hourglass stabilization and triangular shell elements,
- Beam elements with various built-in cross sections,
- 8-Node Flanagan-Belytschko hexahedral elements with hourglass stabilization and hyper-elastic solid elements,
- Various constraint formulations,
- Automatic contact algorithm for node-face, edge-edge, node-edge, node-node contact detection.
- Discrete Element method.

Lattice Discrete Particle Model



Beam Shear Failure

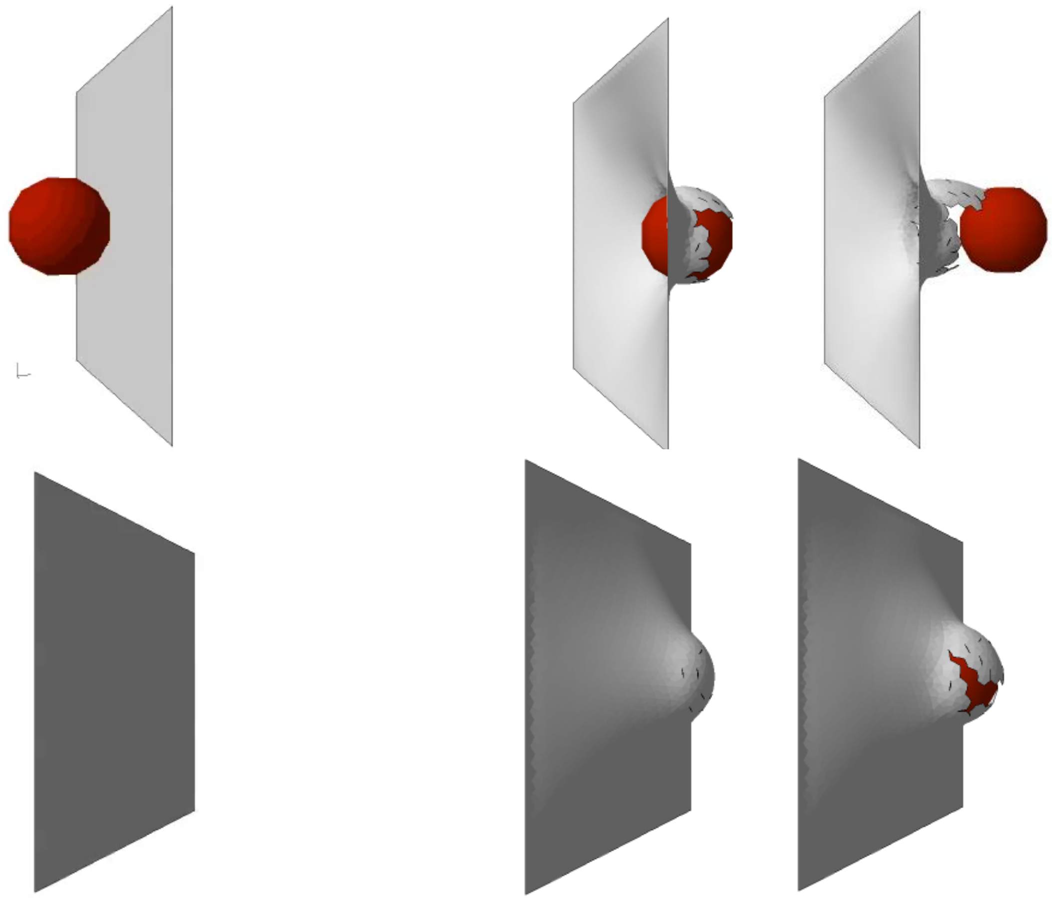
Discrete Fragmentation Algorithm for Solid Components



The weapon case is modeled using conventional 8-node hex elements. Discrete cracks are introduced by performing local remeshing.

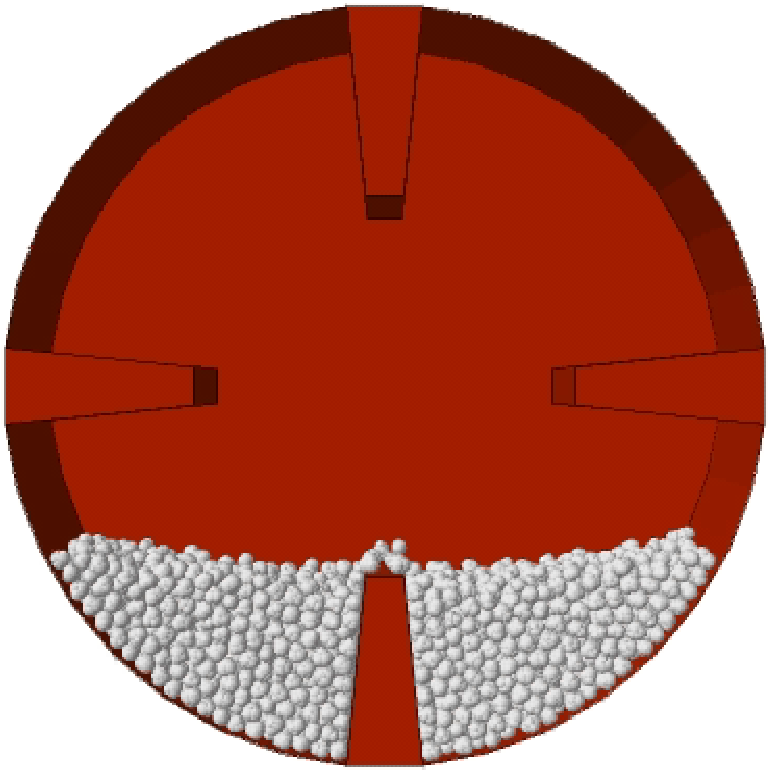
Click on figures to start animations

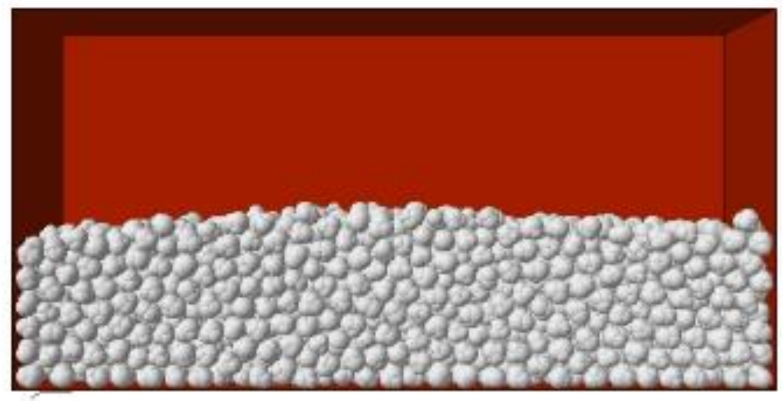
Plate Laceration Due to Fragment Impact



The laceration algorithm inserts small cracks in a continuous mesh based on a local measure of plastic strain and on a Weibull flaw distribution

Realistic Particle Dynamics





Note the jerky motions of the particles inside this rolling container

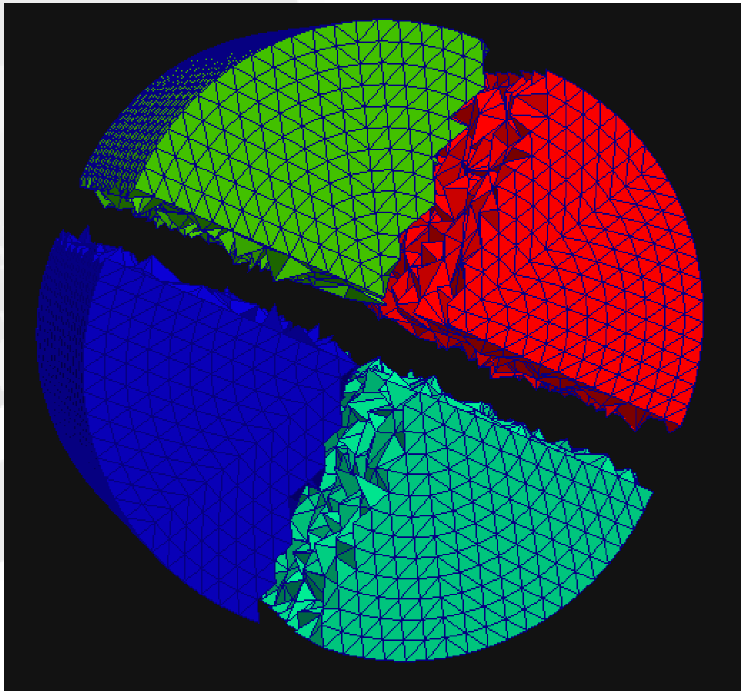
Rotating tumbling mill quickly come to a halt due to macro-particle internal dissipation

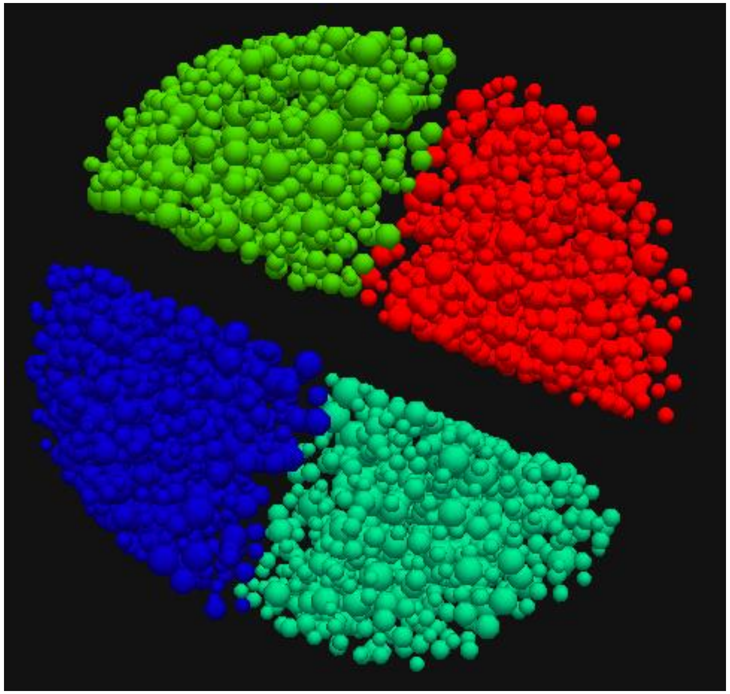
Click on figures to start animations

MPI Domain Decomposition

Domains are visualized using exploded views and different colors

```
PlotList DomainDecomposition {
  Paraview
  TimeInterval 100. s
  ndL Particles {
    DomainDecomposition 1.3
  }
}
```





Try MARS for free at

http://mars.es3inc.com/trymars.php

CONCLUSIONS

Conclusions

- LDPM is a very mature technology that can be confidently used to simulate the behavior of standard and ultra-high performance concrete, without and with fiber reinforcing.
- LDPM shows unprecedented predictive capabilities under a wide variety of loading conditions, both quasi-static and dynamic.
- LDPM is the only approach which has been successfully used to perform predictive multiscale simulations of concrete structures.
- LDPM is ready to tackle practical engineering problems dealing with both long term aging deterioration as well as catastrophic man-made and natural hazards.

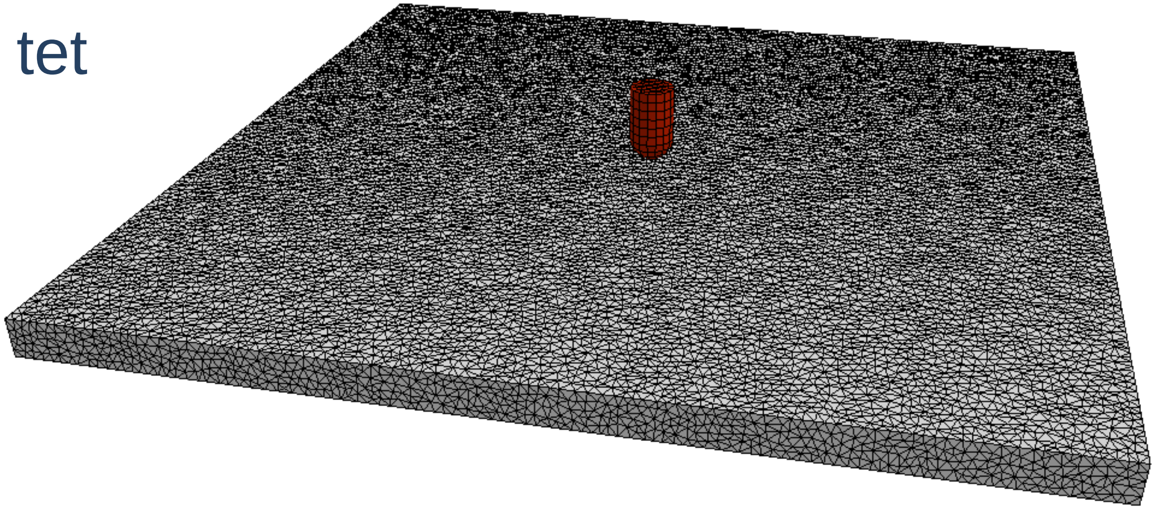
NORTHWESTERN UNIVERSITY

THANK YOU!

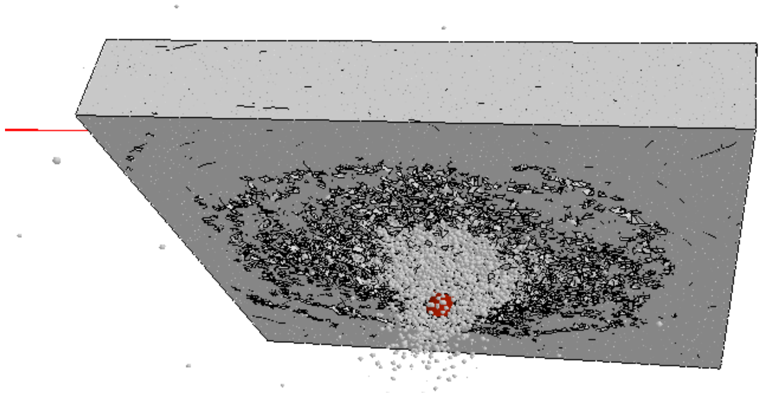
g-cusatis@northwestern.edu
www.cusatis.us

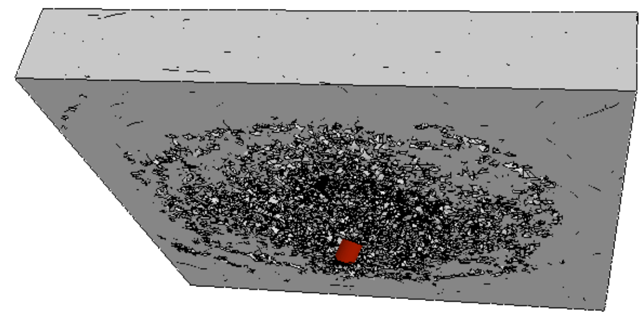
Parallelization of Bullet Impacting FRC Panel

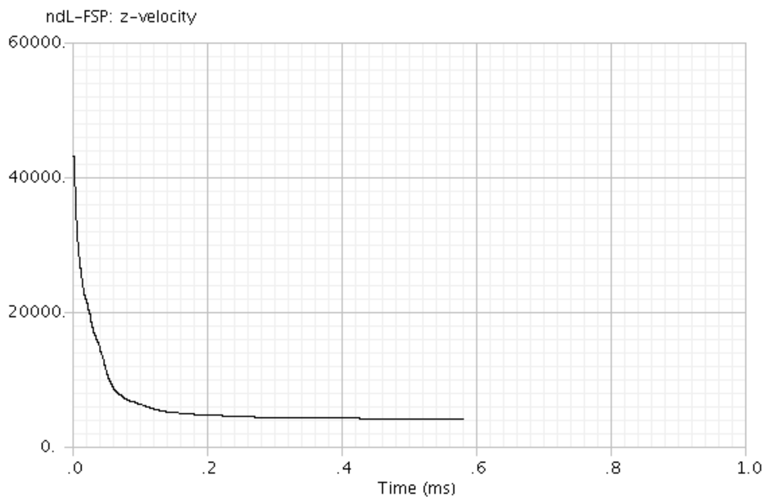
- Panel is model using 3.17 M LDPM tet element
- A geometric tet element requires 40 bytes of memory; a LDPM element requires over 5 Kbytes of memory
- For this problem, recursive bisection employs tet centers as points

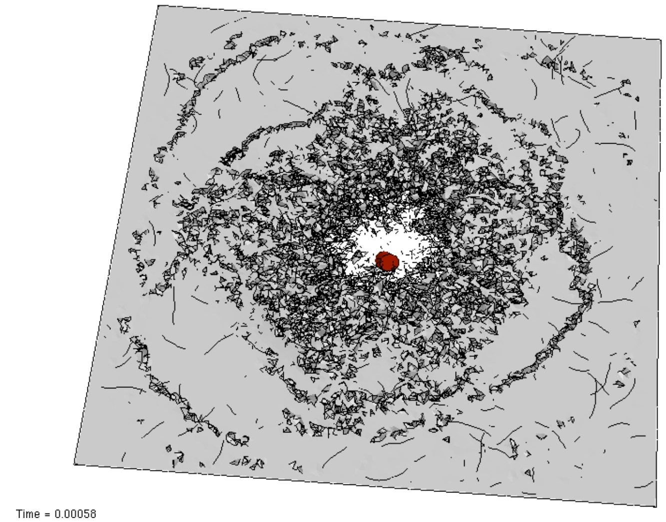


Example of Penetration Results from this Model

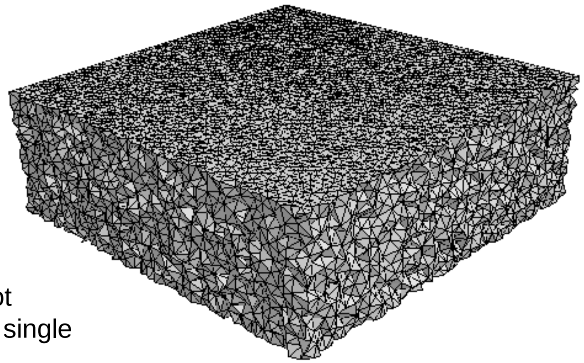


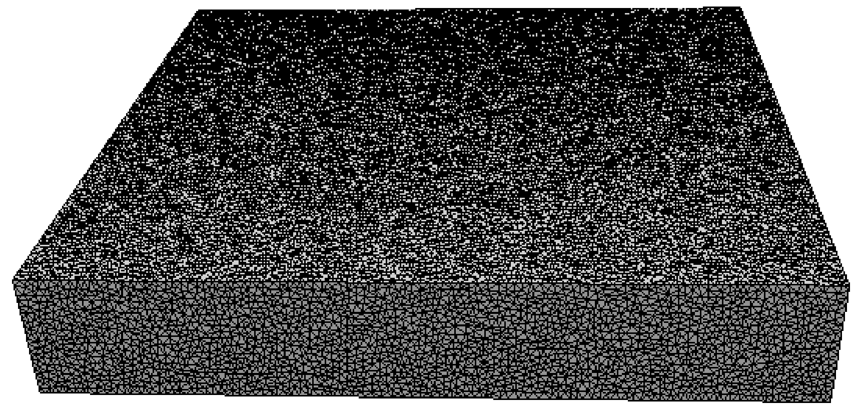




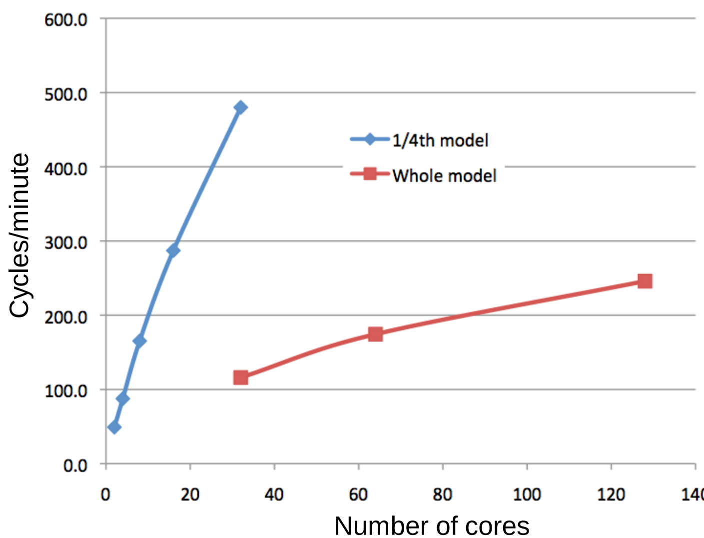


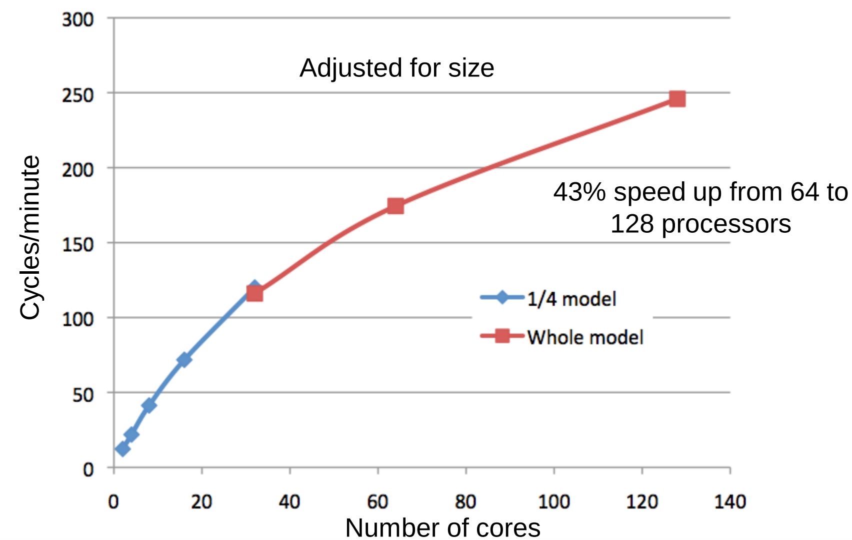
MPI Performance





Whole model does not
fit in the memory of a single
compute node





MARS – Multiscale-multiphysics Analysis of the Response of Structures

http://mars.es3inc.com

Hong Kong, China | Aug 26, 2016

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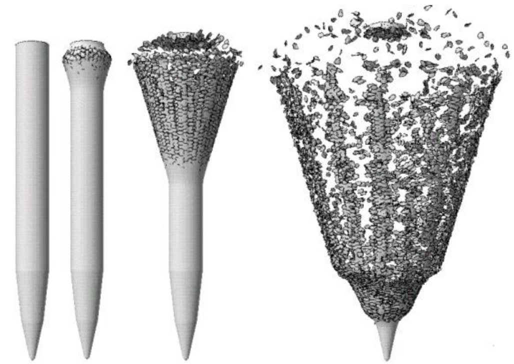
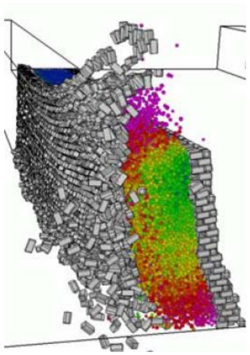
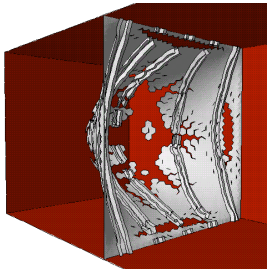


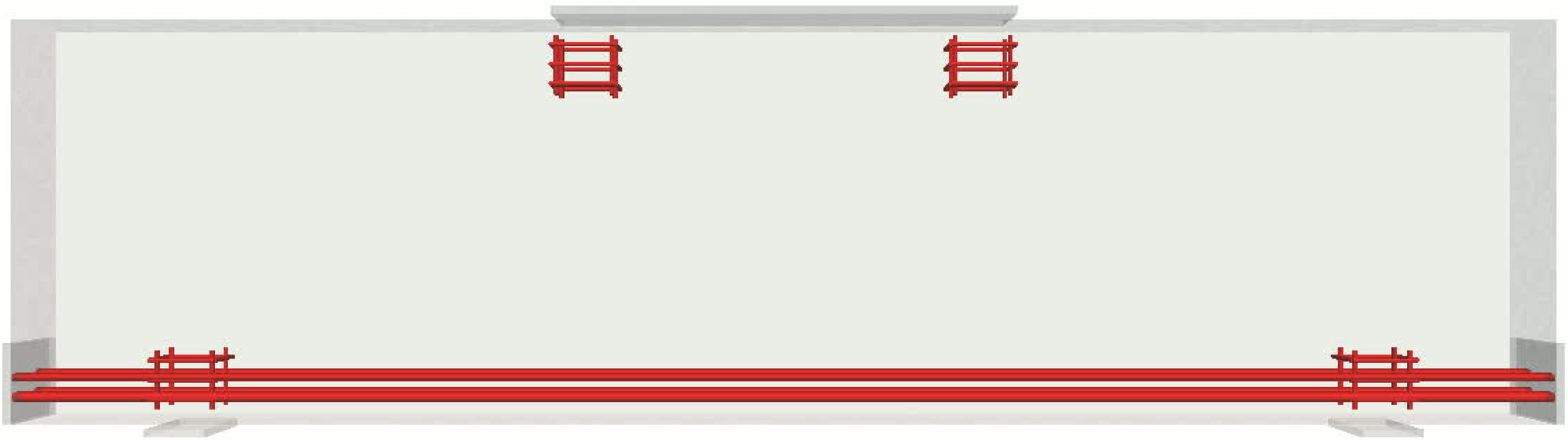




MARS Is a General Purpose Structural Dynamic Code

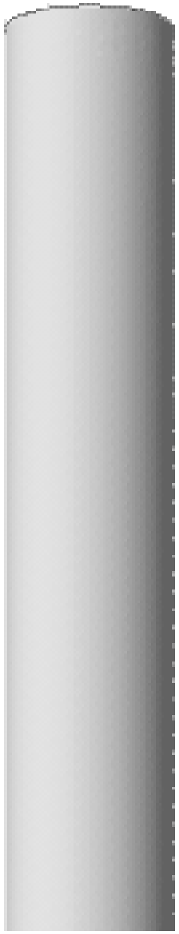
- Lattice Discrete Particle Model (LDPM) for simulations of cementitious materials
- QPH quadrilateral shell elements with physical hourglass stabilization and triangular shell elements,
- Beam elements with various built-in cross sections,
- 8-Node Flanagan-Belytschko hexahedral elements with hourglass stabilization and hyper-elastic solid elements,
- Various constraint formulations,
- Automatic contact algorithm for node-face, edge-edge, node-edge, node-node contact detection.
- Discrete Element method.

Lattice Discrete Particle Model



Beam Shear Failure

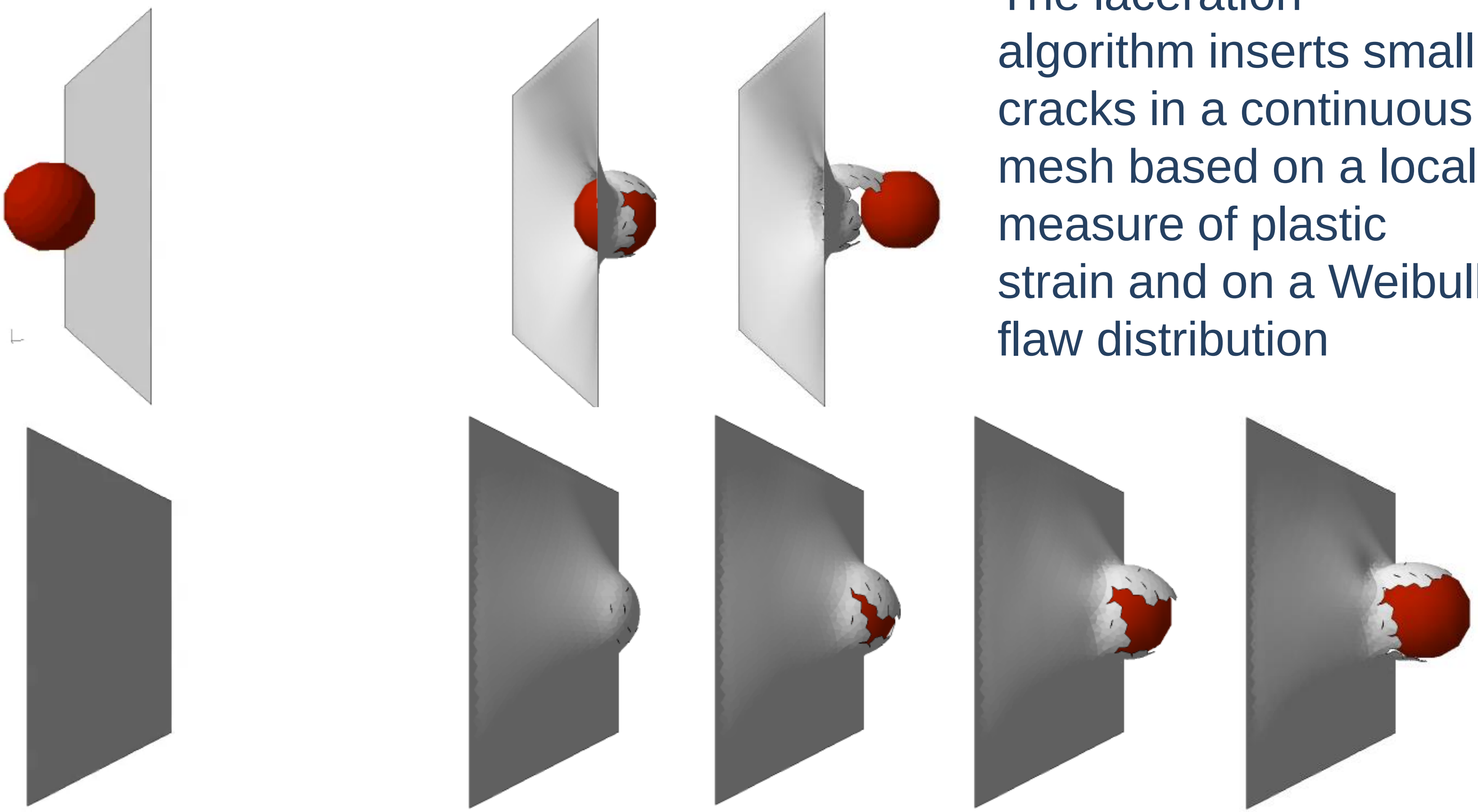
Discrete Fragmentation Algorithm for Solid Components



The weapon case is modeled using conventional 8-node hex elements. Discrete cracks are introduced by performing local remeshing.

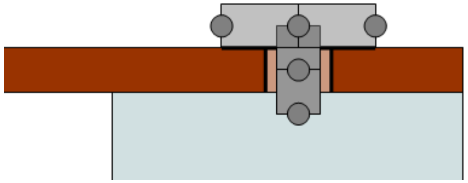
Click on figures to start animations

Plate Laceration Due to Fragment Impact



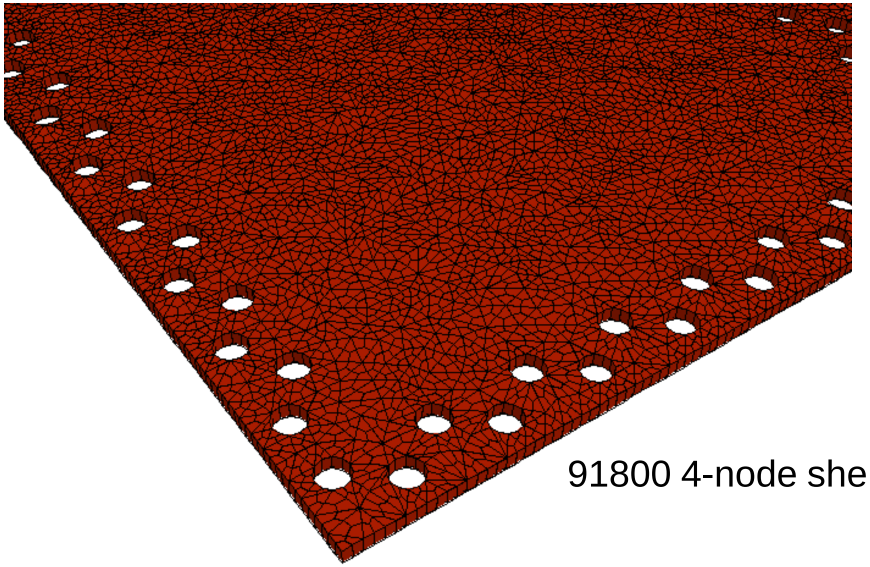
The laceration algorithm inserts small cracks in a continuous mesh based on a local measure of plastic strain and on a Weibull flaw distribution

Exploded View of Plate, Reactive Structure, and Bolts

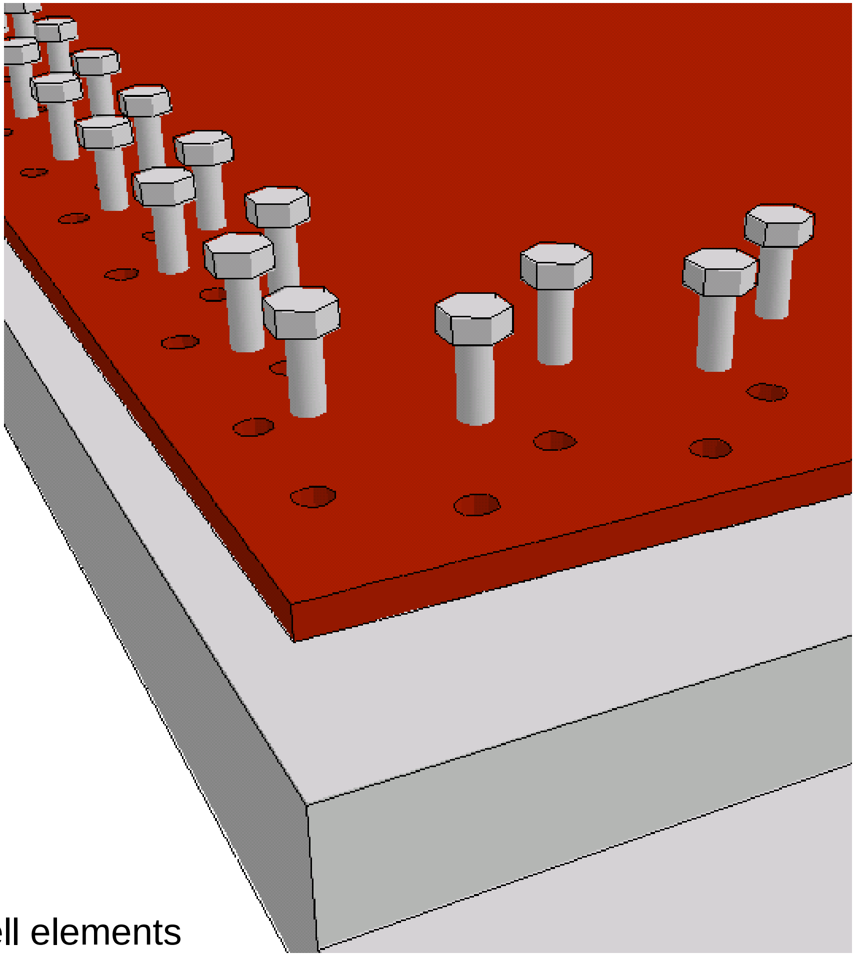


Bolts are modeled using 2 beam elements for the stem and three 4-node shells for the head. Stem can fail under tensile and shear loads.

Components interact using contact elements. Pre-stress is applied to the bolts.

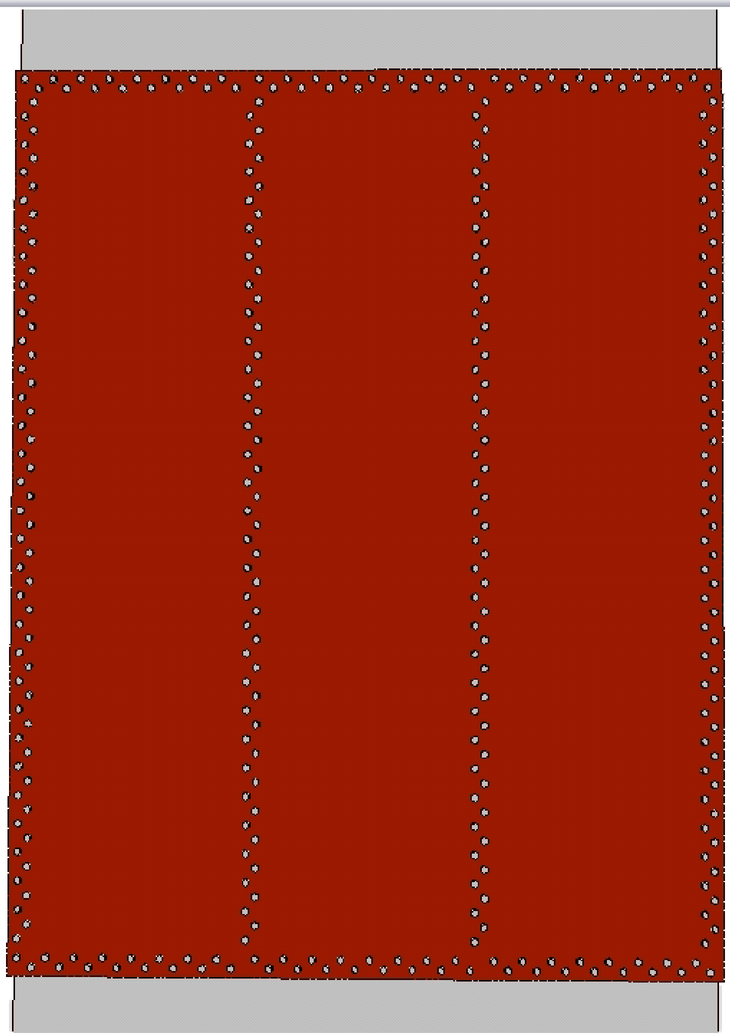


91800 4-node shell elements

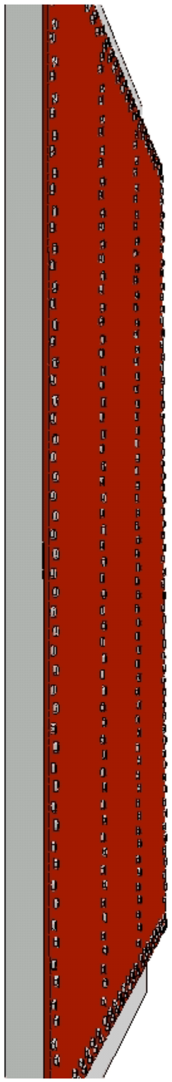


See animations in next page

Shell Laceration
Bolt Failures



Click on figures to start animations

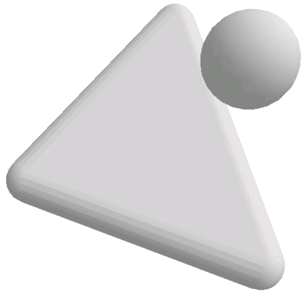


Ability of modeling model complexity

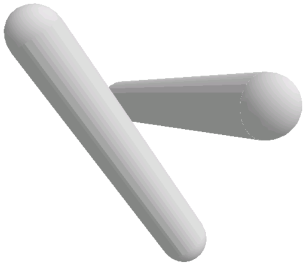
Contact Detection Algorithms in the *MARS* code

The *MARS* contact detection algorithm has the following features:

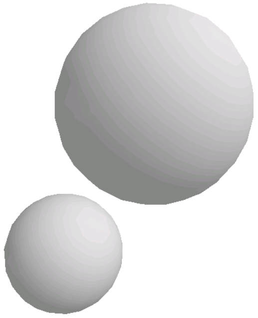
- Arbitrary contacts between face, edges, and particles
- Automatic contact detection with dynamic memory allocation
- One object can interact with multiple other objects at the same time
- Shared contact models: penalty, damping, friction, rolling resistance (similar to material models)



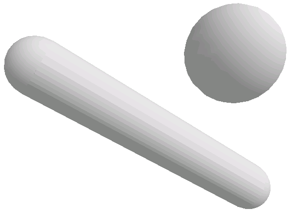
Node/Face Contact



Edge-Edge Contact



Node-Node Contact



Node-Edge Contact

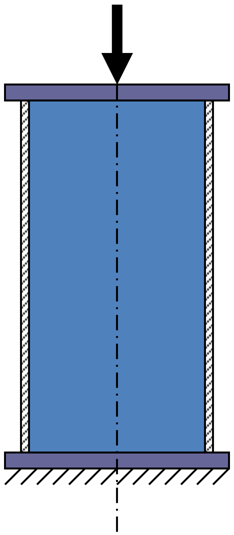
Vertical Compression Buckling of Cylindrical Shell

The top edge of an aluminum cylinder resting on a rigid surface is pushed down causing the cylinder to crush

Triangular shell elements

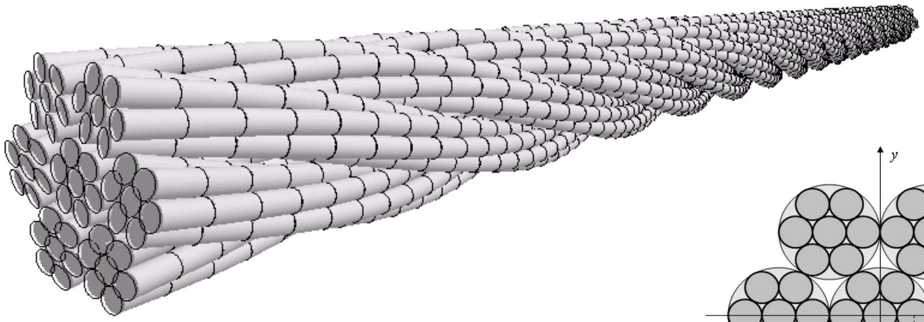
no imperfections ←

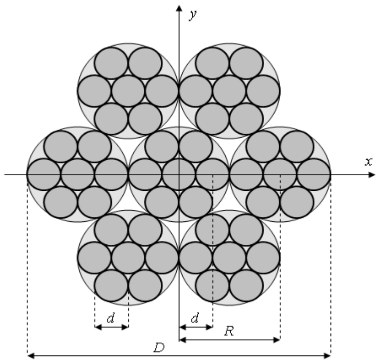
slight surface imperfections →

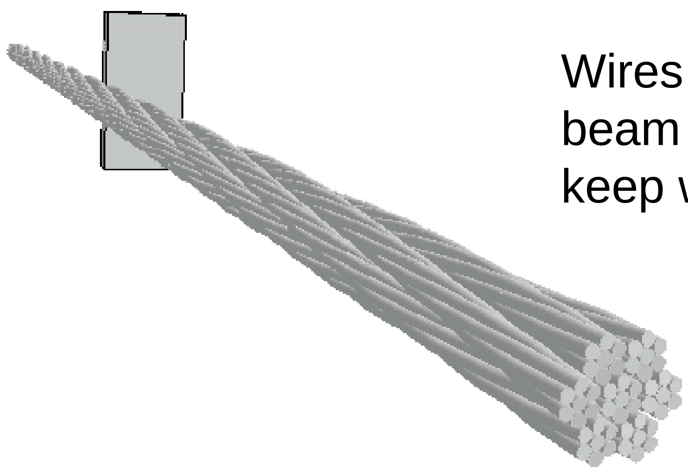


Click on figures to start animations

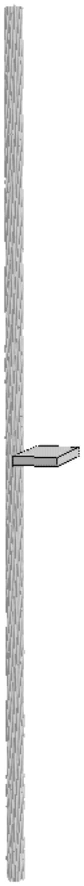
Cable Dynamics







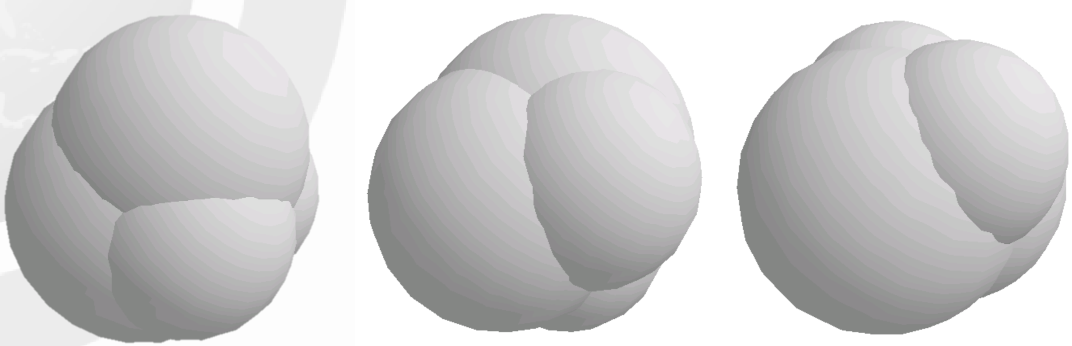
Wires are modeled using strings of beam elements. Edge-edge contacts keep wires from crossing each other.



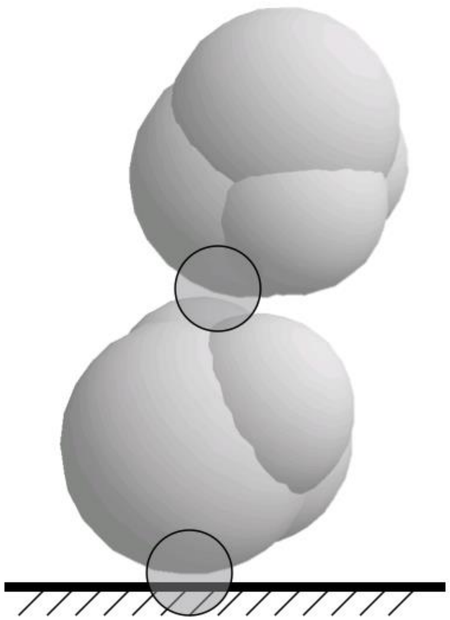
Click on figures to start animations

Discrete Element Method for Modeling Granular Materials

- Soil regions are modeled as random distributions of spherical or non-spherical particles (Discrete Element Method, DEM)
- DEM regions are perfectly integrated with the Finite Element regions of the model.
- Interactions between particles and finite elements employ various types of contact conditions.

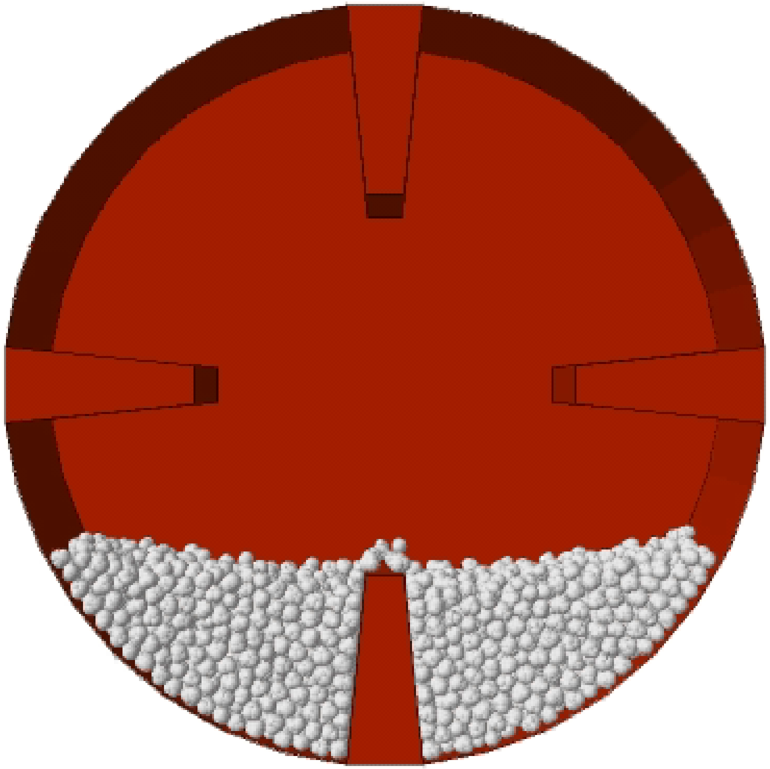


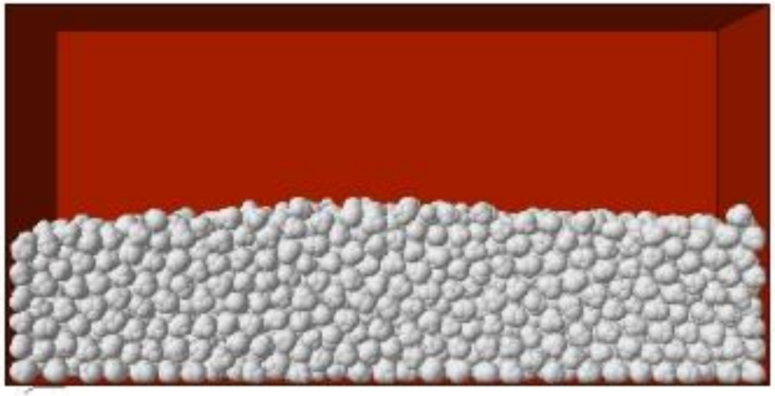
Random shapes of non-spherical macro-particles



Simple contact conditions

Realistic Particle Dynamics



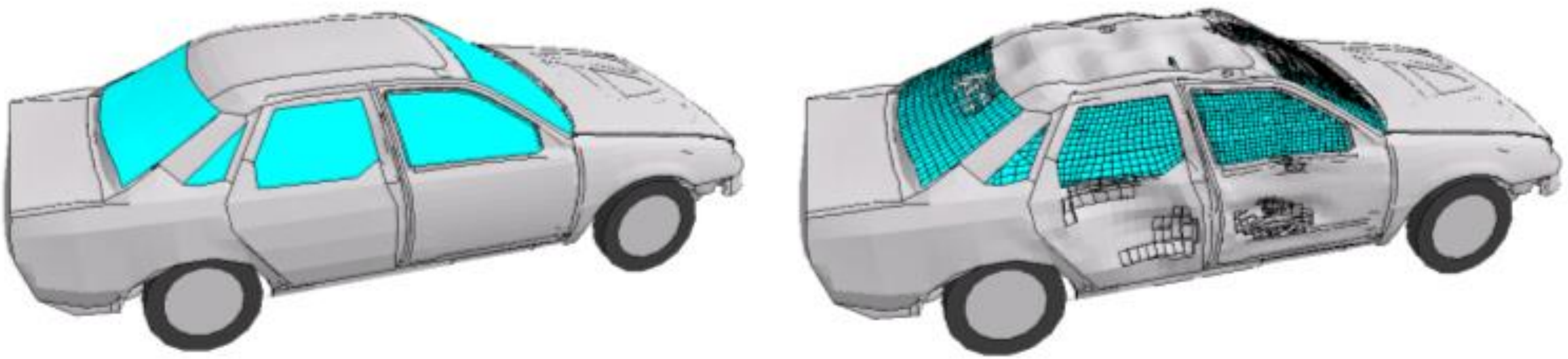


Note the jerky motions of the particles inside this rolling container

Rotating tumbling mill quickly come to a halt due to macro-particle internal dissipation

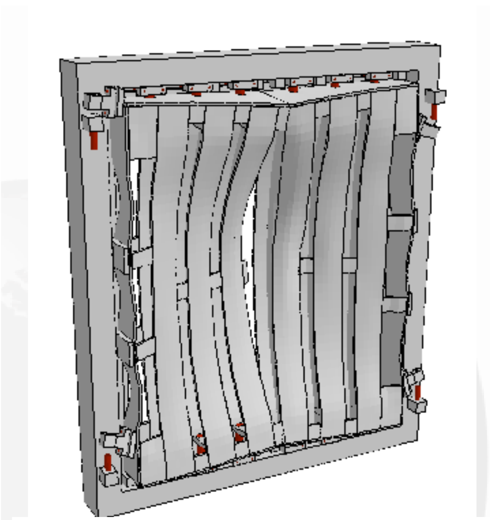
Click on figures to start animations

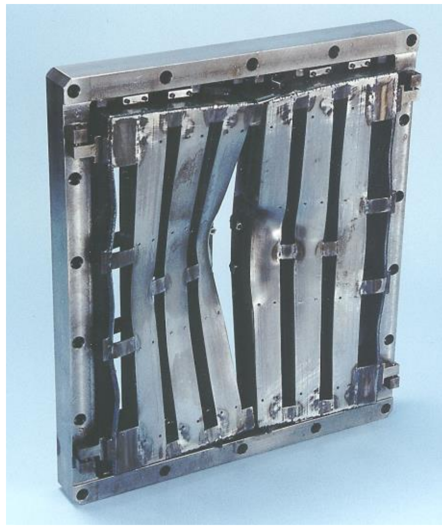
Vehicle Subjected to Explosion



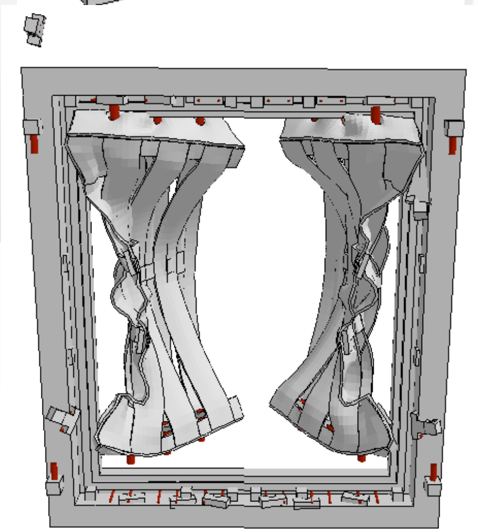
Model of a Ford Taurus (developed by GWU) subjected to external charge loads.

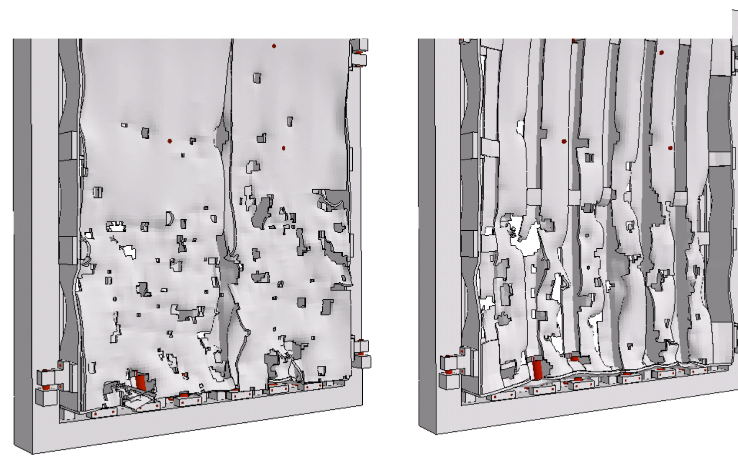
Protective Door Subjected to Blast Loads





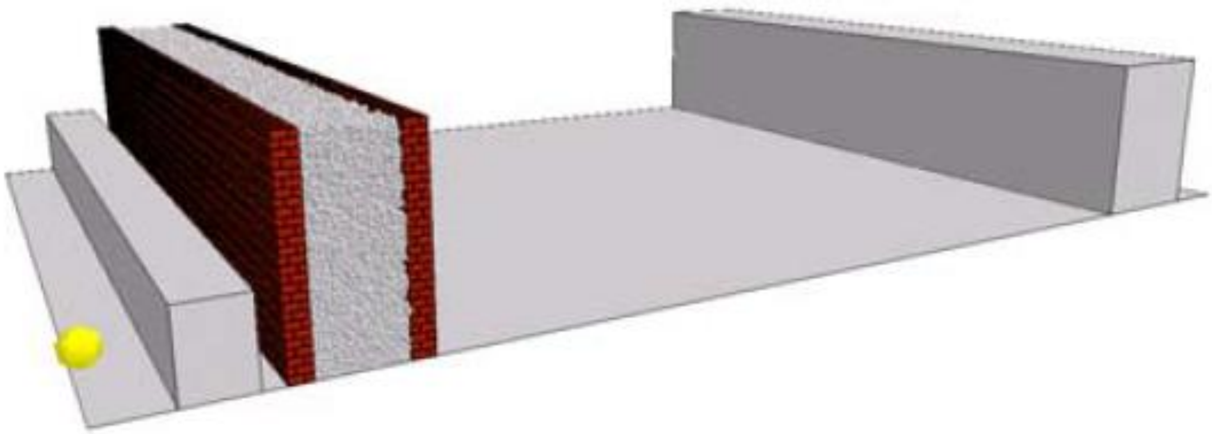
These simulations were performed coupling MARS to a CFD solver



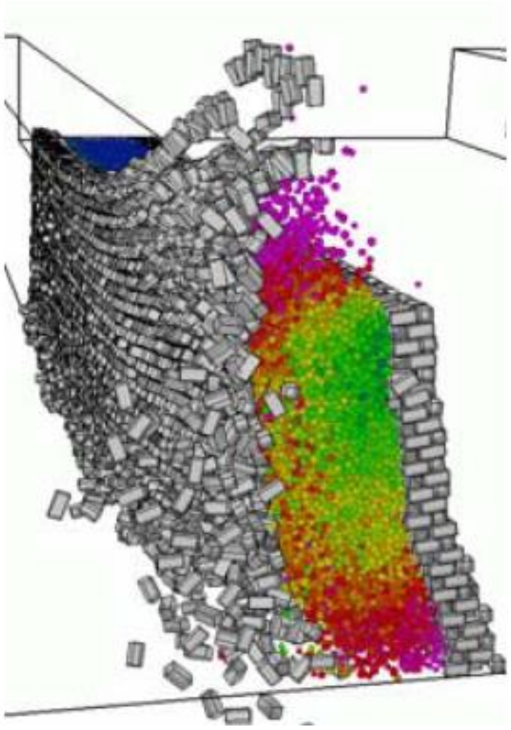


Predictive simulations with blast and fragments

Sandwich Brick Wall Subjected to Blast Loads

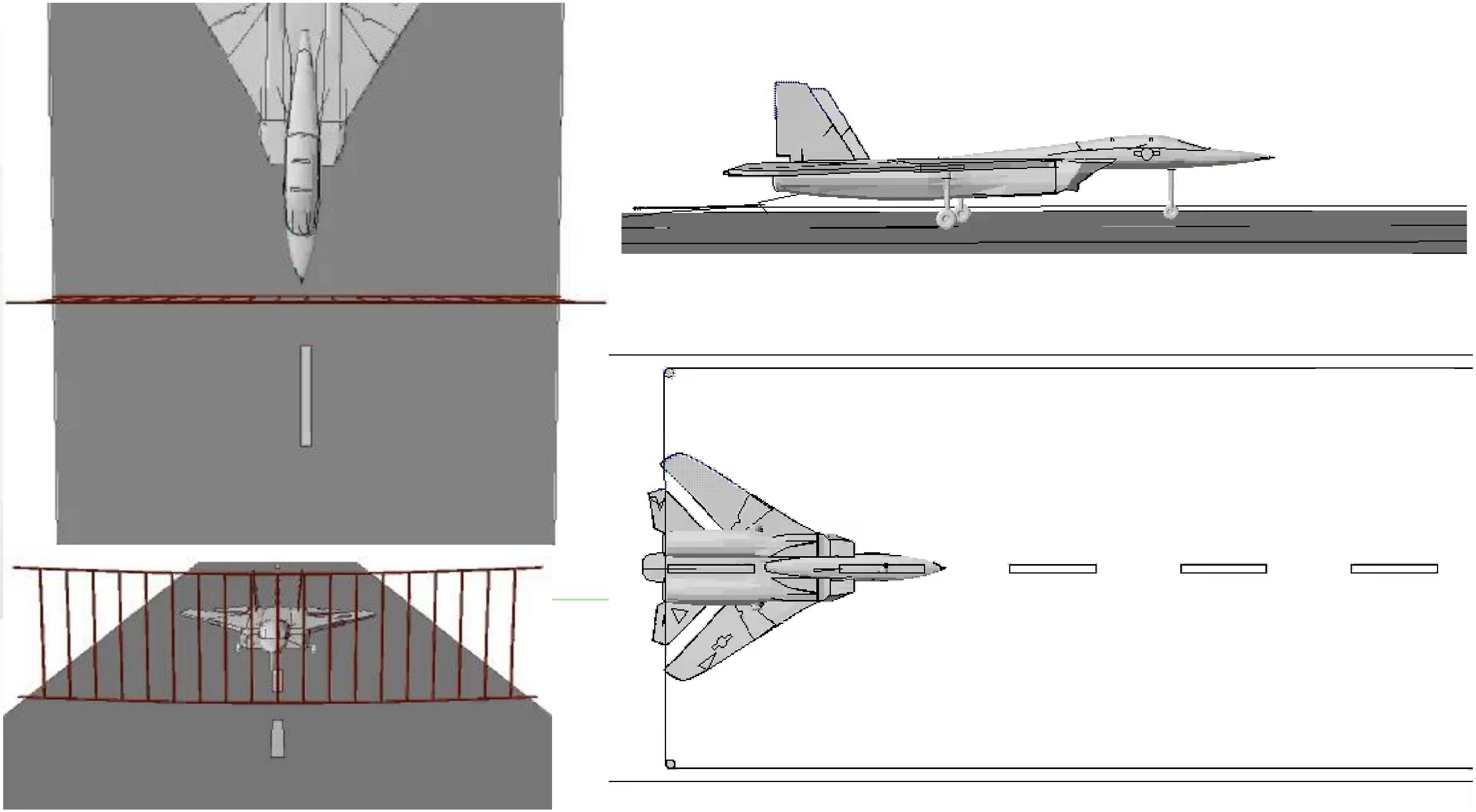


MARS coupled to a CFD solver



Sandwich wall consisting of soil trapped between two brick walls. The wall is subjected to blast loads that propel bricks and soil particles.

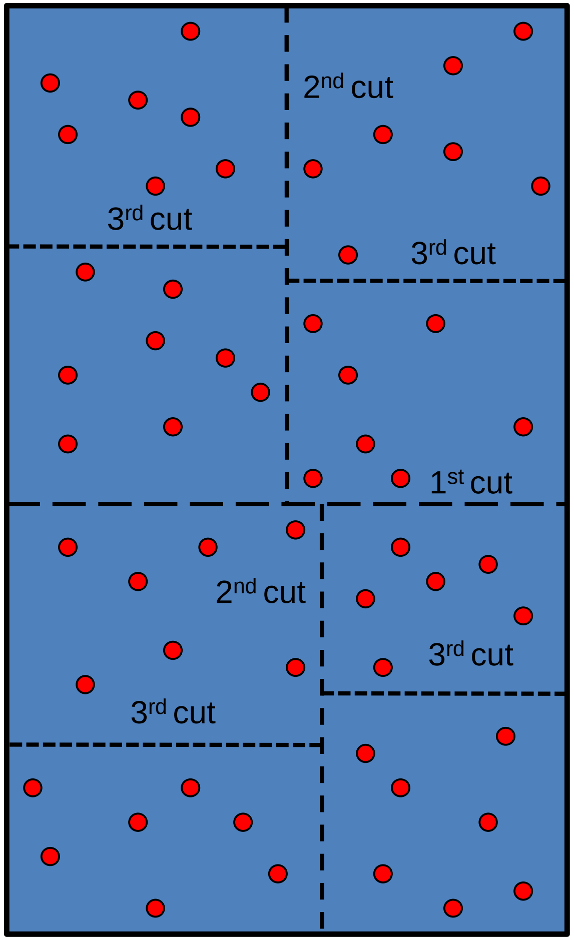
Simulations of Aircraft Arresting Systems



MPI PARALLELIZATION

MARS Employs Recursive Bisection for Domain Decomposition

- Turn most computationally expensive objects into points.
- Decompose space into N bins (domain decomposition) containing equal number of points by recursively splitting the initial bounding bin.
  - At the boundaries, domains are extended to infinity so that any object, no matter where it is located, can be uniquely placed in one of the domains
  - Assign all other objects (contacts included) to domains based on spatial location

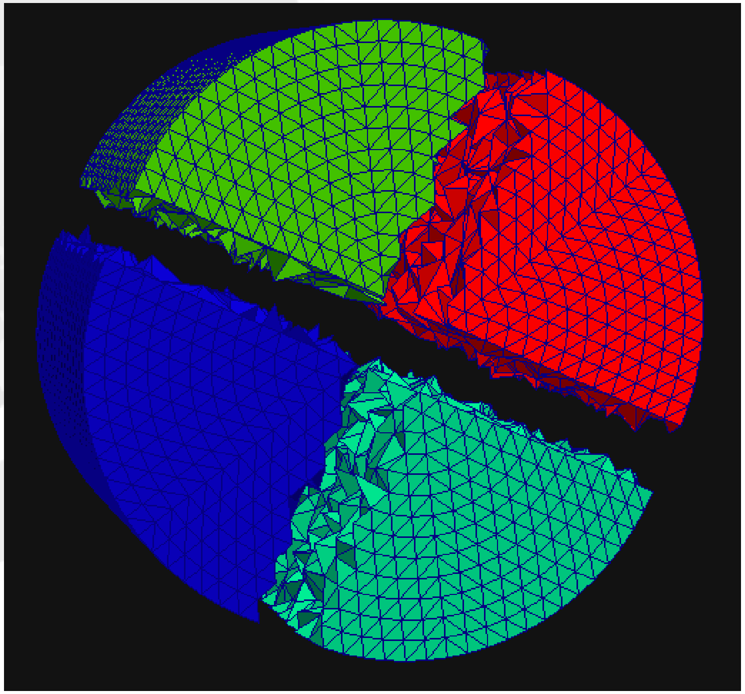


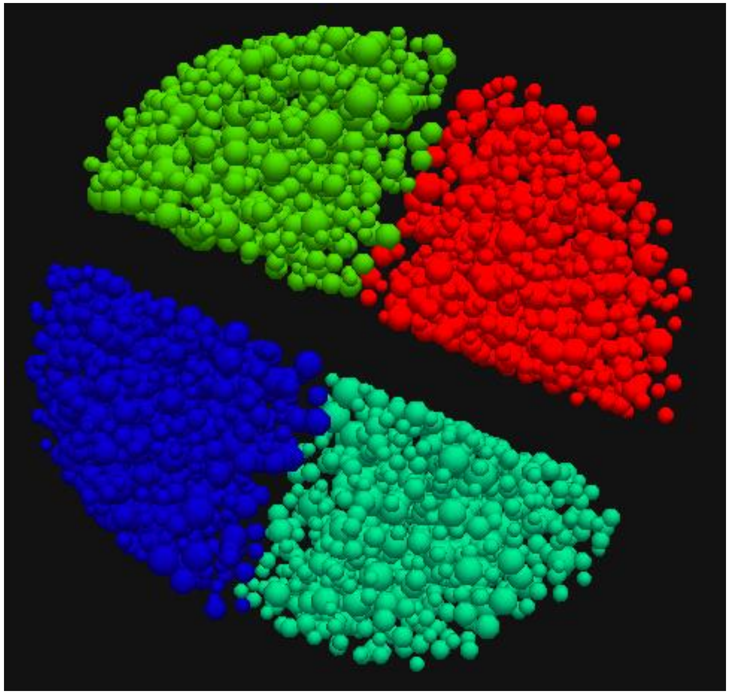
ES3

Visualization of MPI Domain Decompositions

Domains are visualized using exploded views and different paints

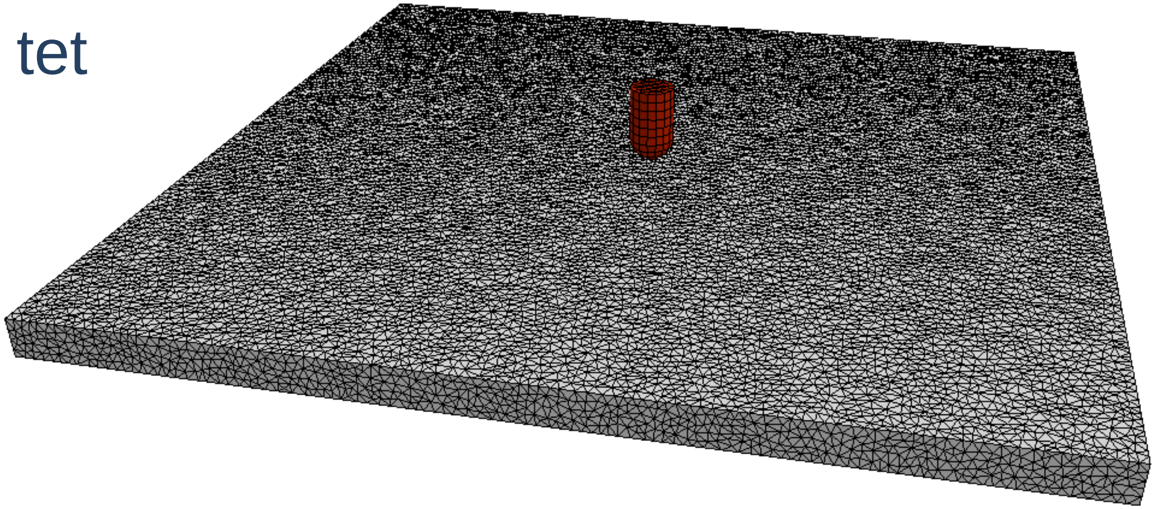
```
PlotList DomainDecomposition {
  Paraview
  TimeInterval 100. s
  ndL Particles {
    DomainDecomposition 1.3
  }
}
```



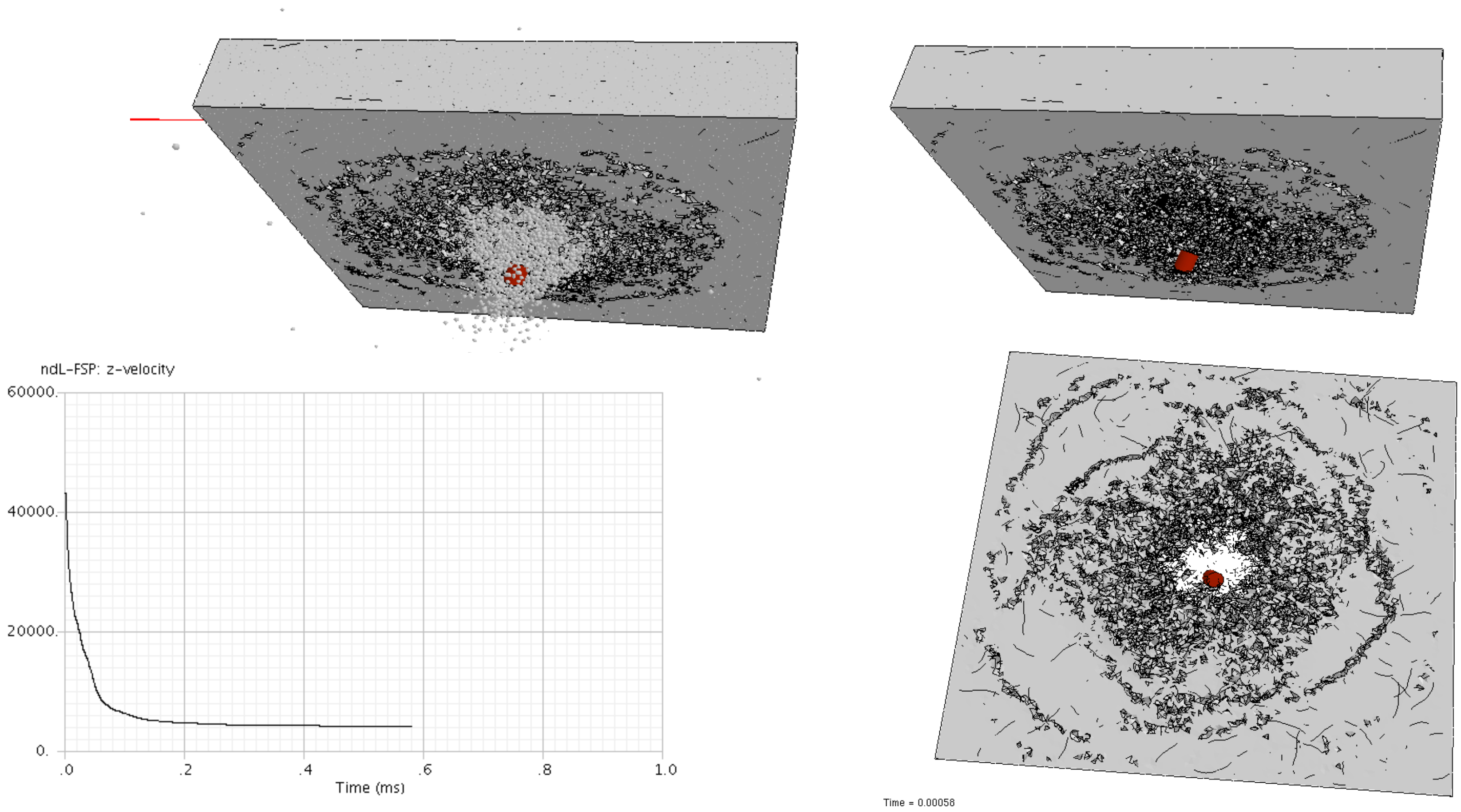


Parallelization of Bullet Impacting FRC Panel

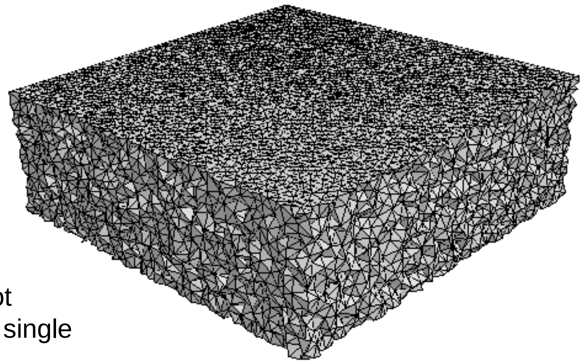
- Panel is model using 3.17 M LDPM tet element
- A geometric tet element requires 40 bytes of memory; a LDPM element requires over 5 Kbytes of memory
- For this problem, recursive bisection employs tet centers as points

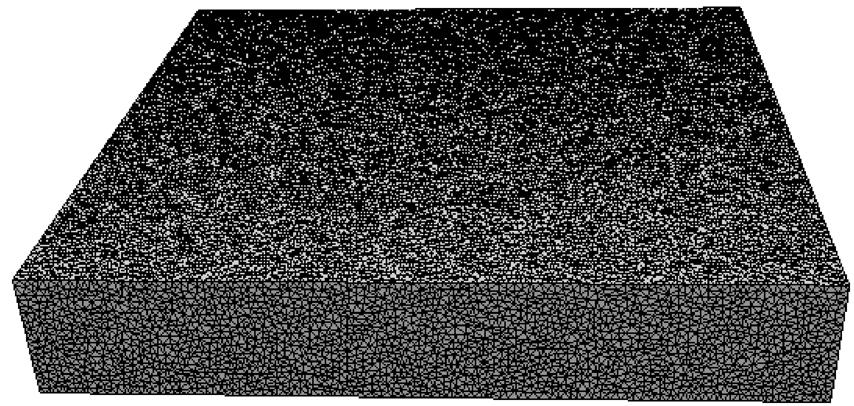


Example of Penetration Results from this Model

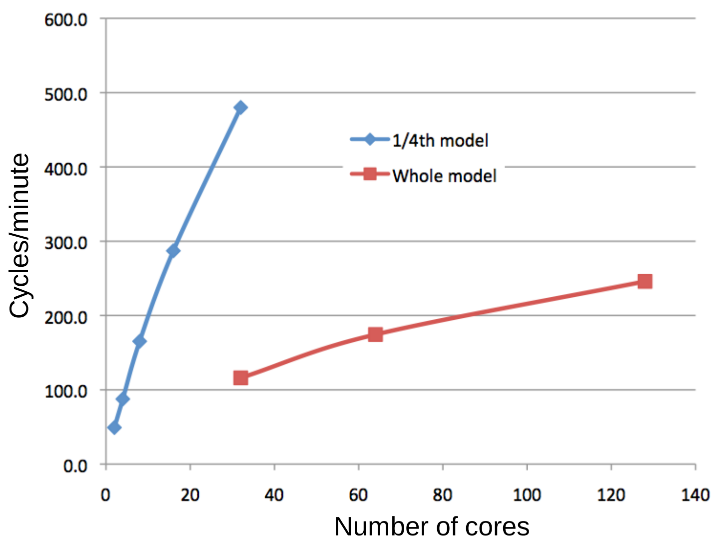


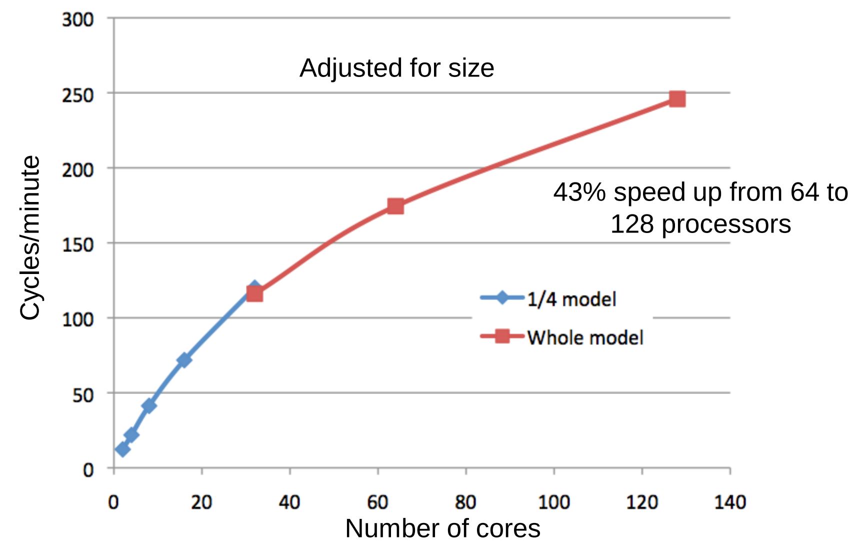
MPI Performance





Whole model does not
fit in the memory of a single
compute node





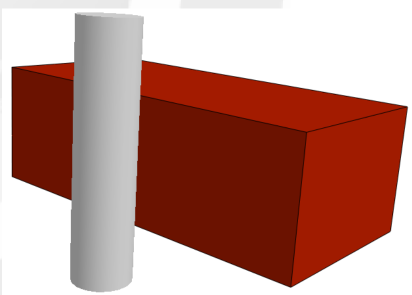
Try MARS for free at

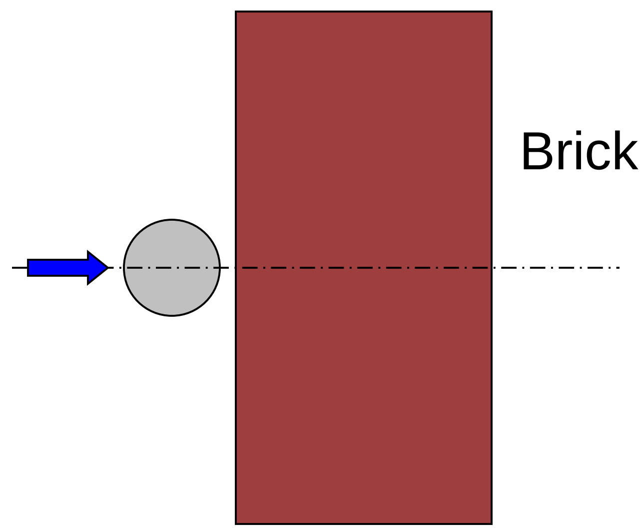
http://mars.es3inc.com/trymars.php

Parametric Study of Fragmentation

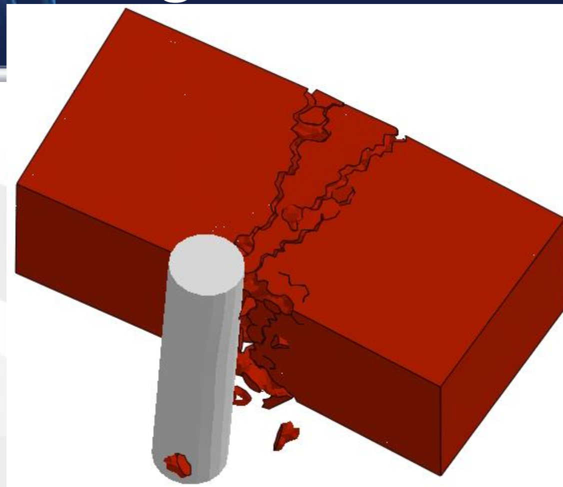
- Impact of a steel cylindrical rod against a quasi-brittle brick
- The objective is to study fragmentation processes
- Various velocities and masses of the cylinder are considered

M = 0.72 Kg

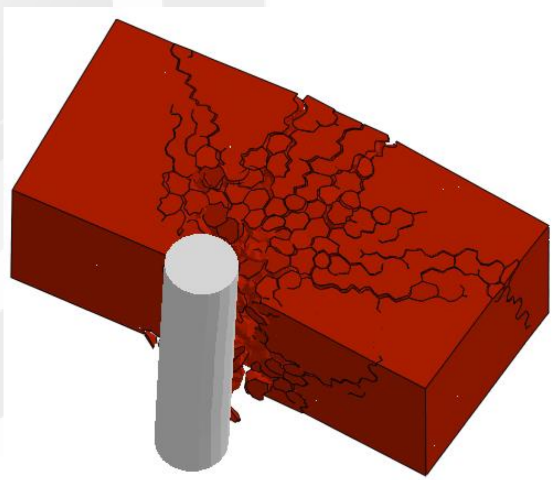




of Fragments Increases with Velocity

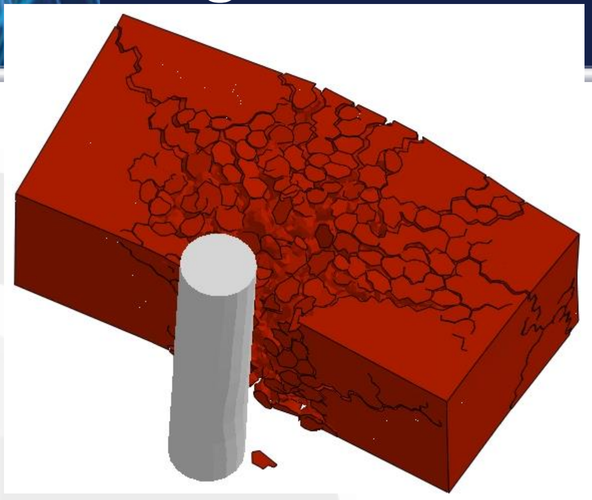


V=400 in/s

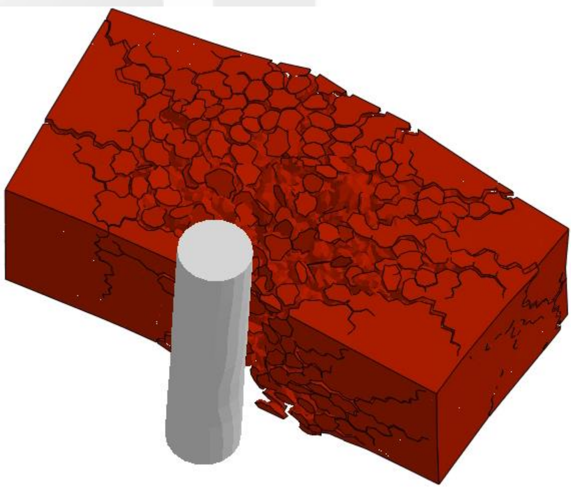


V=800 in/s

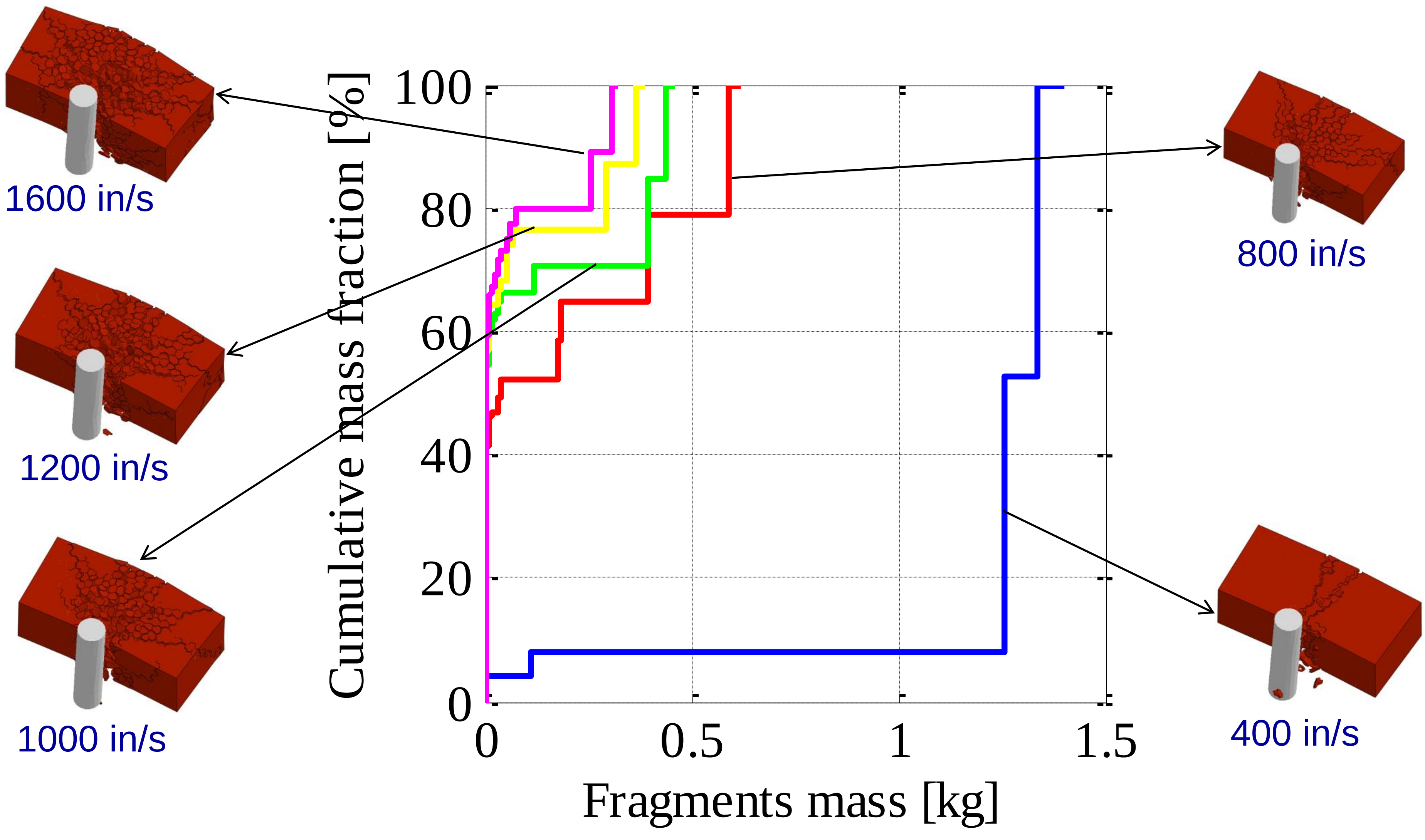
of Fragments Increases with Velocity



V=1200 in/s

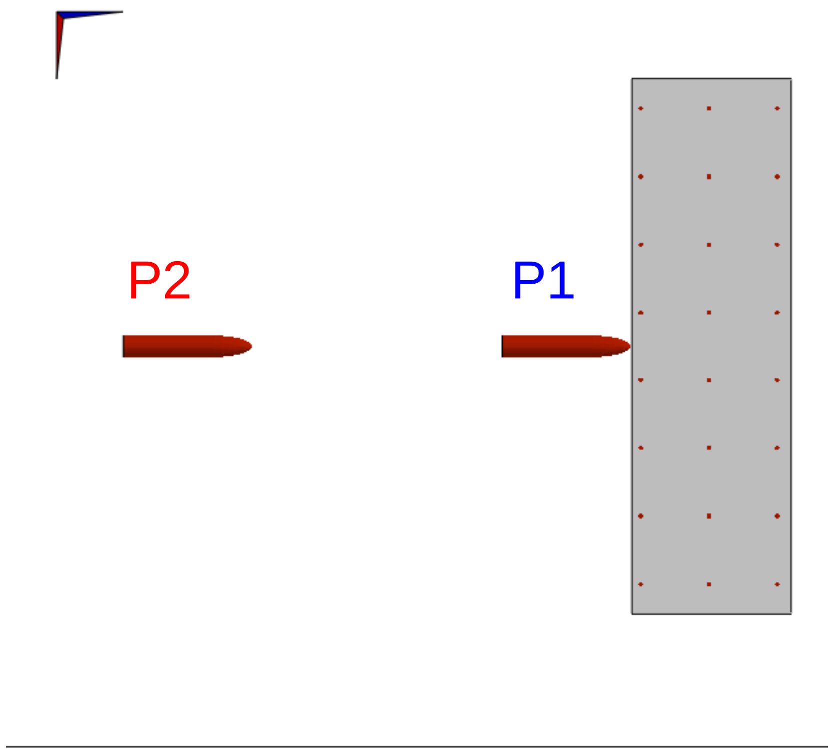


V=1600 in/s

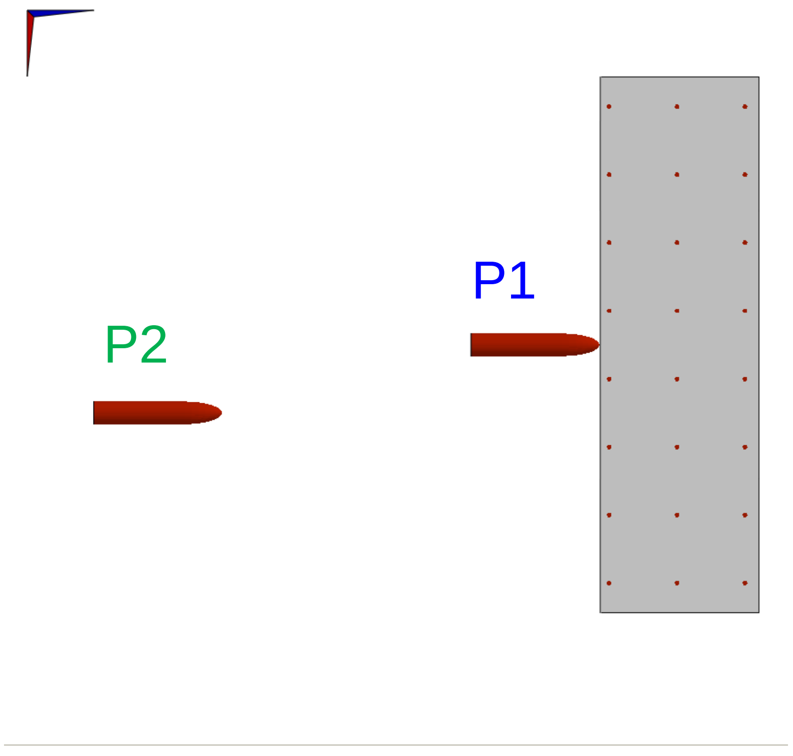


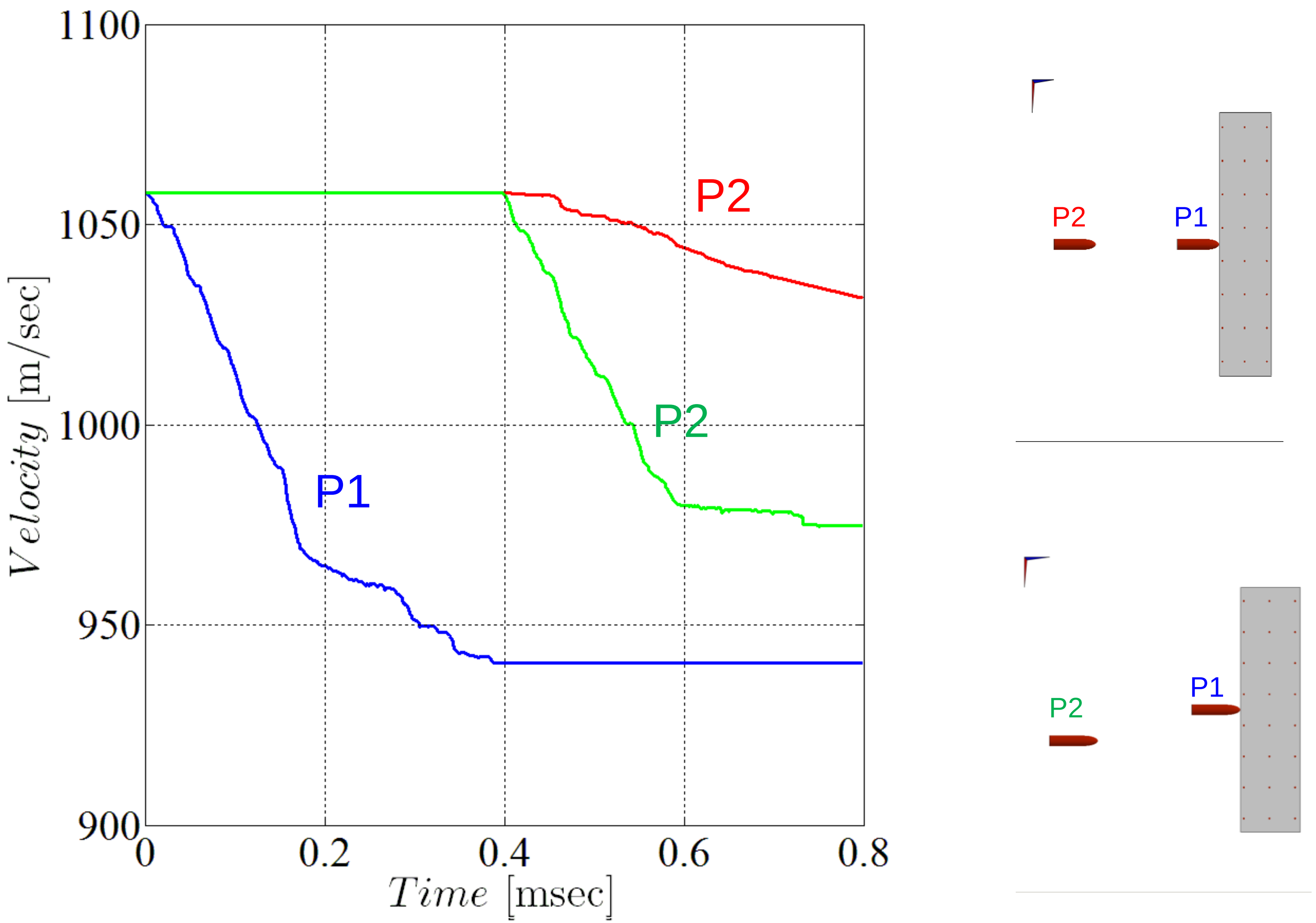
1600 in/s
1200 in/s
1000 in/s
800 in/s
400 in/s
Cumulative mass fraction [%]
100
80
60
40
20
0
0
0.5
1
1.5
Fragments mass [kg]

Case # 1: Centered Hits



Case # 2: Offset Hits




1100
1050
1000
950
900
0
0.2
0.4
0.6
0.8
Velocity [m/sec]
Time [msec]
P1
P2
P2
P2
P1
P2
P1